ACCRS: Algebra I.1-3

Algebra I.1: Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.
Example: We define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.

Algebra I.2: Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Algebra I.1: Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

Mastered:

Students can explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. Example: We define $5^{1//3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1//3})^3$ must equal 5. Rewrite expressions involving radicals and rational exponents using the properties of exponents; and explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

Present:

Students will write a description of the rational and irrational numbers between 0 and 1.

Going Forward:

Students will investigate the real number system which consists of all rational and irrational numbers.

Present and Going Forward Vocabulary:

Rational and irrational numbers, exponents

Career Connections:

Business Manager, Computer Scientist, Engineer

Advanced Understanding & Activity (Alternate activity): (Student page is located in Appendix A.)

Numerical Relationships

Student Instructions: Create an advice column, self-help book, or talk show segment that describes the relationship between the rational and irrational numbers between 0 and 1. Be sure to note the number in each group, examples of each, and how they get along. Be creative and imaginative.

Literature Connections/Resources:

You Tube: http://www.youtube.com/watch?v=QIoVPtbEUjw

ACCRS: Algebra I.4-6

Algebra I.4: Use units as a way to understand problems and to guide the solution of multistep problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

Algebra I.5: Define appropriate quantities for the purpose of descriptive modeling.

Algebra I.6: Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Mastered:
Students can use units as a way to understand problems and to guide the solution of multistep problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays; define appropriate quantities for the purpose of descriptive modeling; and choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Present:
Students will convert (using unit multipliers) the distance from the Earth to Mars which is between 54.6×10^{6} km and 401×10^{6} km to miles using 2.54 cm per inch in the conversion. Also explain why the different distances from Earth to Mars exists.

$$\frac{54.6 \times 10^{6}\text{ km}}{1} \times \frac{1000\text{m}}{1\text{km}} \times \frac{100\text{cm}}{1\text{m}} \times \frac{2.54\text{cm}}{1\text{ in}} \times \frac{12\text{ in}}{1\text{ ft}} \times \frac{5280\text{ft}}{1\text{ mile}}$$

Repeat for 401×10^{6} km.

Going Forward:
Students will find other planets that have varying distances from earth due to orbit.

Present and Going Forward Vocabulary:
Unit multipliers, conversions

Career Connections:
Marketing, Computer Systems Analyst, Real Estate Broker

Advanced Understanding & Activity (Alternate activity): (Student page is located in in Appendix A)

Planetary Conversions

Student Instructions: Using unit multipliers, convert the distance from Earth to Mars, which is between 54.6×10^{6} km and 401×10^{6} km to miles, using 2.54 cm per inch in the conversion. Then create a written document, PowerPoint, or video explaining to elementary students how the distance from Earth to Mars can change. Include your data.

Literature Connections/Resources:
http://donnayoung.org/math/unit_multipliers.htm

ACCRS: Algebra I.7-8

Algebra I.7: Interpret expressions that represent a quantity in terms of its context.
a. Interpret parts of an expression such as terms, factors, and coefficients.
b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *Example*: Interpret $P(1+r)^n$ as the product of P and a factor not depending on P.

Algebra I.8: Use the structure of an expression to identify ways to rewrite it. *Example*: See $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.

Mastered:
Students can interpret expressions that represent a quantity in terms of its context.
a. Interpret parts of an expression such as terms,

Present:
Students will investigate the Berry method of factoring trinomials. (See link below) Write a “That’s Good, That’s Bad” chain story about factoring trinomials.

Going Forward:
Students will look into factoring polynomials with four terms.

factors, and coefficients.

b. Interpret complicated expressions by viewing one or more of their parts as a single entity.

Example: Interpret $P(1+r)^n$ as the product of P and a factor not depending on P, and use the structure of an expression to identify ways to rewrite it.

Example: See $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.

Present and Going Forward Vocabulary:

Expressions, Berry method, Guess and Check method, Factor by Grouping method, trinomials

Career Connections:

Teacher/Educator, Statistician, Engineer, Mathematician

Advanced Understanding & Activity (Alternate activity): (Student page is located in Appendix A.)

That's Good, That's Bad!

Student Instructions: Investigate the Berry Method of factoring trinomials. Compare and contrast it to the Guess and Check method and the Factor by Grouping method. Then write a **"**That's Good, That's Bad" chain story about factoring trinomials.

Scenario: Multiple methods to factor trinomials

Literature Connections/Resources:

- http://vimeo.com/15536626
- Cuyler, Margery and Catrow, David. That's Good! That's Bad! NY: Henry Holt & Co. 1993.

ACCRS: Algebra I.9

Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

a. Factor a quadratic expression to reveal the zeros of the function it defines.

b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

c. Determine a quadratic equation when given its graph or roots.

d. Use the properties of exponents to transform expressions for exponential functions.

Example: The expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.

Mastered:

Students can choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

a. Factor a quadratic expression to reveal the zeros of the function it defines.

b. Complete the square in a quadratic expression to

Present:

Students will be given two numbers and asked to demonstrate at least six different uses of the combination.

Going Forward:

Students will increase their ability to factor using different strategies.

reveal the maximum or minimum value of the function it defines. c. Determine a quadratic equation when given its graph or roots. d. Use the properties of exponents to transform expressions for exponential functions. Example: The expression 1.15^{t} can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.		

Present and Going Forward Vocabulary:
Roots, addition, subtraction, ordered pairs

Careers:
Computer Information Systems Manager, Actuary, Engineer

Advanced Understanding & Activity (Alternate activity): (Student page is located in Appendix A.)

Math Manipulation

Student Instructions: Given the numbers 6 and -3, show at least six different ways these numbers can be manipulated or used as a part of a math problem. One of these should be a quadratic equation. Think of the many, varied, and unusual ways you can combine these numbers. When would these different manipulations be used or what professions would use the different manipulations?

Literature Connections/Resources:
http://answers.yahoo.com/question/index?qid=20110714115828AAlIUAe

ACCRS: Algebra I.10
Understand that polynomials form a system analogous to the integers; namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Mastered:	Present:	Going Forward:
Students can understand that polynomials form a system analogous to the integers; namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.	Students will use The Thinker Keys – Commonality to compare and contrast Integers and Polynomials	Students will determine when closure would be important.

Present and Going Forward Vocabulary:
Closure in Math, polynomials, addition, subtraction, multiplication

Career Connections:
Computer Information Systems Manager, Mathematician, Budget Analyst

Advanced Understanding & Activity (Alternate activity): (Student page is located in Appendix A.)

Thinker Keys

Student Instructions: Using Thinker Keys choose to complete one or more of the following:

Commonality: Compare and Contrast Integers and Polynomials with regard to operations and closure.

Disadvantages: What are the disadvantages of using polynomials?

What If?: What if closed sets were not allowed? How would this change mathematics?

Picture: Transform this shape into something related to integers and polynomials. You may not turn this shape into a number or equation.

Different Uses: What are all the different and unusual uses of polynomials? Be creative!

Literature Connections/Resources:

Wikipedia: http://en.wikipedia.org/wiki/Closure_(mathematics)

ACCRS: Algebra I.11-.14

Algebra I.11: Create equations and inequalities in one variable, and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

Algebra I.12: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

Algebra I.13: Represent constraints by equations or inequalities, and by systems of equations and/or inequalities and interpret solutions as viable or non-viable options in a modeling context. *Example:* Represent inequalities describing nutritional and cost constraints on combinations of different.

Algebra I.14: Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *Example*: Rearrange Ohm's law $V = IR$ to highlight resistance R.

Mastered:

Students can create equations and inequalities in one variable, and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential function;* create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales; represent constraints by equations or inequalities, and by systems of equations and/or inequalities and interpret solutions as viable or non-viable options in a modeling context.

Example: Represent inequalities describing nutritional and cost constraints on combinations of different foods; rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

Present:

Students will take the literal equation challenge. Prior to doing this they may practice solving literal equations at: http://regentsprep.org/REgents/math/ALGEBRA/AE4/litPrac.htm.

Going Forward:

Students will look over formulas used to solve problems and get information in retail businesses at: http://retail.about.com/od/retailingmath/a/retail_formulas.htm.

Example: Rearrange Ohm's law $V = IR$ to highlight resistance *R; and* solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Present and Going Forward Vocabulary:
Formulas, literal equations

Career Connections:
Mathematician, Statistician, Teacher/Educator

Advanced Understanding & Activity (Alternate activity): (Student page is located in Appendix A.)

Literal Equation Challenge

Student Instructions: Formulas (Literal Equations) that deal with measurement through higher-mathematics are used everywhere. It is imperative to be able to manipulate these equations. Practice this skill at http://regentsprep.org/REgents/math/ALGEBRA/AE4/litPrac.htm.
When you are ready, take the literal equation challenge at http://regentsprep.org/REgents/math/ALGEBRA/AE4/Tliterals.htm.

Literature Connections/Resources:
http://www.ehow.co.uk/how_8665248_solve-simple-literal-equations.html

ACCRS: Algebra I.15
Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

Mastered:	Present:	Going Forward:
Students can **e**xplain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	Students will be presented with the proof of 1=2. They will be asked to determine what is incorrect about the proof.	Students will explore other situations in math where using valid math postulates and theorems could provide someone with the wrong answer.

Present and Going Forward Vocabulary:
Argument, postulate, theorem, definition

Career Connections:
Engineer, Architecture, Contractor

Advanced Understanding & Activity (Alternate activity): (Student page is located in Appendix A.)

What's Wrong With This Proof?

Student Instructions: Analyze the Algebraic proof below. Then write a "That's Good! That's Bad" chain story using the scenario provided**.**

In the chain story, you will write and illustrate the chain of events set off by the scenario to show the positive and negative situations caused by each event. You may use additional sheets of paper in order to complete your story.

Remember that the story must relate to and demonstrate an understanding of the proof. Feel free to interject imagination and humor in your story.

Given:	$a = b$
Multiply both sides by a	$a^2 = ab$ Multiplication Property of Equality
Subtract b^2 from both sides	$a^2 - b^2 = ab - b^2$ Subtraction Property of Equality
Factor both sides	$(a-b)(a+b) = b(a-b)$ Distributive Property of Equality
Divide both sides by	$(a-b)$ $\frac{(a-b)(a+b)}{(a-b)} = \frac{b(a-b)}{(a-b)}$ Division Property of Equality
$a+b=b$	
Substitute *a* for *b* because *a*=*b*	$a+a=a$
Combine like terms	$2a=a$
Divide both sides by *a*	$\frac{2a}{a} = \frac{a}{a}$
	2 = 1 Division property of equality

Scenario: According to the Division Property of Equality, two now equals one (2-1). Everything is now buy one, get one free, because two items equal one item. Oh, that's good! No, that's bad!

Literature Connections/Resources:

- Reasoning and Proof in High School Textbooks from the USA**:** http://tsg.icme11.org/document/get/282.
- Cuyler, Margery and Catrow, David. That's Good! That's Bad! NY: Henry Holt & Co. 1993.

ACCRS: Algebra I.16-.17

Algebra I.16: Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Algebra I.17: Solve quadratic equations in one variable.

a. Use the method of completing the square to transform any quadratic equation in *x* into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.

b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square and the quadratic formula, and factoring as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions, and write them as $a \pm bi$ for real numbers *a* and *b*.

Mastered:

Students can solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Students can solve quadratic equations in one variable.

a. Using the method of completing the square to transform any quadratic equation in *x* into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.

b. Solving quadratic

Present: Project

Students will define the structure of an electronic game with at least eight levels which has to do with solving from simple one step equations to using various method of solving quadratic equations with imaginary roots.

Going Forward:

Students will think of other ways that mathematic procedures could be used in a game format.

equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square and the quadratic formula, and factoring as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions, and write them as $a \pm bi$ for real numbers a and b.		

Present and Going Forward Vocabulary:
Linear equations, completing the square, quadratic formula

Career Connections:
Medical and Health Service Manager, Purchasing Manager, Actuary, Engineer

Advanced Understanding & Activity (Alternate activity): (Student page is located in Appendix A.)

Electronic Games

Student Instructions: Design the structure of an electronic math game with at least eight levels. Many games have math equations to solve in order to progress to the next level. Develop a theme or story for your game. Then develop the equations to be solved at each level. For example, level one could start with solving one step equations. You should use all basic operations in each level. You can increase the difficulty by using integers instead of whole numbers, or rational numbers instead of integers. Have at least one level that deals with literal equations and use the final level to solve quadratic equations having imaginary roots. How challenging and complex can you make your game? You can write out the levels on a piece of paper or use one of the online game sites to develop your game, such as Scratch (http://scratch.mit.edu/), or use PowerPoint with clickable links. Challenge your classmates to play your game.

Literature Connections/Resources:
- http://www.sosmath.com/algebra/solve/solve0/solve0.html
- Scratch: http://scratch.mit.edu/
- YoYo Games: http://www.yoyogames.com/gamemaker/

ACCRS: Algebra I.18-20

Algebra I.18: Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

Algebra I.18: Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Algebra I.18: Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.

Example: Find the points of intersection between the line $y = –3x$ and the circle $x^2 + y^2 = 3$.

Mastered:	Present:	Going Forward:
Students can prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions; solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on	Students will work with a partner and play the two person game for solving systems of equations at http://www.crctlessons.com/systems-of-equations-game.html. Students can play the one player game, but it is just solving equations and not systems. Students will then write a word problem and story that uses systems of equations.	Students will look for things in their life that can be solved by using systems of equations.

pairs of linear equations in two variables; and solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. Example: Find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.

Present and Going Forward Vocabulary:

Systems of equations, graphing, substitution, elimination

Career Connections:

Engineer, Financial Manager, Computer Analyst

Advanced Understanding & Activity (Alternate activity): (Student page is located in Appendix A.)

Shoot the Hoop!

Student Instructions:

1. Find a partner and play the basketball game at http://www.crctlessons.com/systems-of-equations-game.html. If you are really good, try the time challenge.
2. The one player game is not a system of equations but is various levels of solving linear equations. Individually, write a word problem using systems of equations about a day you had when it seemed as though TOO many things were going wrong or right!
3. Write the problem and the answer then write a story to explain the situation. You may wish to draw pictures to illustrate your story.

Literature Connections/Resources:

http://www.crctlessons.com/systems-of-equations-game.html

ACCRS: Algebra I.21-23

Algebra I.21: Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

Algebra I.22: Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

Algebra I.23: Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Mastered:	Present:	Going Forward:
Students can understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line); explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations; include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions; and graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.	Students will play a game requiring knowledge of linear equations and slope located at http://www.mathplayground.com/SaveTheZogs/SaveTheZogs.html They will have to be patient and learn the controls of the game. Once they have done this, they should enjoy this game.	Students will explore additional lessons learned while playing games.

Present and Going Forward Vocabulary:
Systems of equations, graphing solutions, substitution, elimination

Career Connections:
Computer Information Systems Manager, Actuary, Engineer

Advanced Understanding & Activity (Alternate activity): (Student page is located in Appendix A.)

Save the Zogs

Student Instructions: Go to the Web site http://www.mathplayground.com/SaveTheZogs/SaveTheZogs.html and play the graphing game, Save the Zogs. Be patient and take your time as you learn the controls of the game.

Literature Connections/Resources:
http://www.mathplayground.com/SaveTheZogs/SaveTheZogs.html

ACCRS: Algebra I.24-26

Algebra I.24: Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.

Algebra I.25: Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Algebra I.26: Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. [F-IF3]
Example: The Fibonacci sequence is defined recursively by
$f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ for $n \geq 1$.

Mastered:

Students can understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$; use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context; and recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. [F-IF3] Example: The Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ for $n \geq 1$.

Present:

Students will play the function game found at: http://www.math10.com/en/math-games/games/linear-functions/games-functions.html.

Going Forward:

Students will function play an important roll in their lives, taking into consideration their surroundings and the on-going functions.

Present and Going Forward Vocabulary:

Functions, domain, range

Career Connections:

Computer Information Systems Manager, Actuary, Engineer

Advanced Understanding & Activity (Alternate activity): (Student page is located in Appendix A.)

Functionality

Student Instructions: Go to http://www.math10.com/en/math-games/games/linear-functions/games-functions.html and play the function game.

Literature Connections/Resources:

- http://www.math10.com/en/math-games/games/linear-functions/games-functions.html.

ACCRS: Algebra I.27-29

Algebra I.27: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*

Algebra I.28: Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
Example: If the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.

Algebra I.29: Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Mastered:
Students can for a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.* Students can relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
Example: If the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. Students can calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Present:
Students will use the linear function machine at: http://www.shodor.org/interactivate/activities/LinearFunctMachine/ and practice interpreting functions.

Going Forward:
Students will be aware of situations where the domain is restricted in math as well as life.

Present and Going Forward Vocabulary:
Domain, range, function rule

Career Connections:
Computer Information Systems Manager, Actuary, Engineer

Advanced Understanding & Activity (Alternate activity): (Student page is located in Appendix A.)

Frustration with Functions

Student Instructions:

1. Use the linear function machine at http://www.shodor.org/interactivate/activities/LinearFunctMachine/ and practice interpreting at least t

ten functions. Record all of your inputs and outputs.

2. Now write a "That's Good! That's Bad"! chain story using the scenario provided. Write about and illustrate the chain of events to show the positive and negative situations surrounding the scenario about the frustrations of picking a function rule that works with one number but not another number.
Your story must demonstrate an understanding of why different function rules exist. Feel free to insert imagination and humor in your story. You may use additional sheets of paper if necessary.

SCENARIO: John wanted to find an easy rule to solve functions. Oh, that's good! No that's bad

Literature Connections/Resources:

- http://www.shodor.org/interactivate/activities/LinearFunctMachine/
- Cuyler, Margery and Catrow, David. That's Good! That's Bad! NY: Henry Holt & Co. 1993

ACCRS: Algebra I.30-32

Algebra I.30: Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
- a. Graph linear and quadratic functions, and show intercepts, maxima, and minima.
- b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
- c. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

Algebra I.31: Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
- a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
- b. Use the properties of exponents to interpret expressions for exponential functions. Example: Identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, and $y = (1.2)^{t/10}$, and classify them as representing exponential growth and decay.

Algebra I.32: Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
Example: Given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

Mastered:
Students can graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
- a. Graph linear and quadratic functions, and show intercepts, maxima, and minima.
- b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
- c. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric

Present:
Students will identify the origin of functions in mathematics and note those given credit for innovations in the first years. They will determine what other mathematical concepts were found around the time that functions were introduced.

Going Forward:
Students will determine if other functions are used in their everyday lives. Students will determine if they know anyone who works on commission.

functions, showing period, midline, and amplitude. Students can write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. b. Use the properties of exponents to interpret expressions for exponential functions. Example: Identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, and $y = (1.2)^{t/10}$, and classify them as representing exponential growth and decay. Students can compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). Example: Given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.		

Present and Going Forward Vocabulary:
Graphic functions, maximum, minimum, exponential

Career Connections:
Historian, Mathematician, Engineer, Computer Information Systems Manager, Actuary

Advanced Understanding & Activity (Alternate activity): (Student page is located in Appendix A.)

Who's the Man (or Woman)?

Student Instructions: Write a report about the mathematician that is given credit for introducing functions. Include other accomplishments of this mathematician. Who were other mathematicians from the same time period and what were their accomplishments? Were any of these discoveries related to each other? If yes, how were they related to each other? Were there other discoveries in mathematics during this time period? Explain why these contributions are significant to our study of functions today.

Literature Connections/Resources:
Wikipedia: http://en.wikipedia.org/wiki/Leonhard_Euler

ACCRS: Algebra I.33-36

Algebra I.33: Write a function that describes a relationship between two quantities.
a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
b. Combine standard function types using arithmetic operations.
Example: Build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.

Algebra I.34: Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

Algebra I.35: Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

Algebra I.36: Find inverse functions.
a. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse, and write an expression for the inverse.
Example: $f(x) = 2x^3$ or $f(x) = (x+1)/(x-1)$ for $x \neq 1$.

Mastered:

Students can write a function that describes a relationship between two quantities.
a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
b. Combine standard function types using arithmetic operations.

Example: Build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.

Students can write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

Students can identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

Students can find inverse functions.
a. Solve an equation of the

Present:

Students will use technology to compare the units digit of result of the numbers 1-10 raised to the 2nd, 3rd, 4th, 6th and 8th powers. Then they will answer questions and use inductive reasoning to determine if large numbers could be the result of raising a number to an even power.

Going Forward:

Students will choose other calculations and inspect the last digit to see if it is indicative of a pattern.

form $f(x) = c$ for a simple function f that has an inverse, and write an expression for the inverse.
Example: $f(x) = 2x^3$ or $f(x) = (x+1)/(x-1)$ for $x \neq 1$.

Present and Going Forward Vocabulary:
Excel, frequencies, roots, patterns, inverse functions

Career Connections:
Computer Information Systems Manager, Actuary, Engineer

Advanced Understanding & Activity (Alternate activity): (Student page is located in Appendix A.)
EXCELlent Comparisons!

Student Instructions: Use Excel to make a table and determine the results for $F(x) = x^2$ and $f(x) = x^3$ using a domain of 1-10. Answer the following questions:

1. Make a frequency table listing the frequency in the range each of the digits 0-9 appear as the last digit of the squares. Add columns and find the frequencies of the digits for $f(x)= x^4$, x^6, x^8.
2. Answer the following without further use of technology.
   a. Speculate what the frequencies are for x^{10}.
   b. Speculate what the frequencies are for x^{26}.
   c. What patterns do you see forming?
   d. Could the fourth root of 20736 be a rational number?
   e. Could the fourth root of 4098 be a rational number?

Literature Connections/Resources:
http://www.mathsisfun.com/numberpatterns.html

ACCRS: Algebra I.37-40

Algebra I.37: Distinguish between situations that can be modeled with linear functions and with exponential functions.
- a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
- b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
- c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

Algebra I.38: Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

Algebra I.39: Observe, using graphs and tables, that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

Algebra I.40: Interpret the parameters in a linear or exponential function in terms of a context.

Mastered:
Students can distinguish between situations that can be modeled with linear functions and with exponential functions.
a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over

Present:
Students will compare two methods of problem solving and then write their own. They will use a THINK FAST activity to test their method.

Going Forward:
Students will use their problem solving method and refine it as they finish this year in school.

equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. Students can construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table); observe, using graphs and tables, that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. Students can interpret the parameters in a linear or exponential function in terms of a context.		

Present and Going Forward Vocabulary:
Linear functions, exponential functions, problem solving

Career Connections:
Computer Information Systems Manager, Actuary, Engineer

Advanced Understanding & Activity (Alternate activity): (Student page is located in Appendix A.)

THINK FAST

Student Instructions: Compare and contrast the problem solving methods suggested on these two Web sites. http://www.math.utah.edu/~pa/math/polya.html and http://www.problemsolving.net/wrksheet.html Now THINK FAST Follow the directions at each letter. Write your answers as quickly as possible on a separate sheet of paper.

MATH

Make your own set of methods.

Apply them to Math word problems and/or science problems. (Show at least two problems one a linear function and one an exponential function and how you use you method to solve them.)

Tell how you intend to use technology in solving problems.

How can you use your methods to solve problems not related to math and science?

Literature Connections/Resources:

- http://www.math.utah.edu/~pa/math/polya.html
- http://www.problemsolving.net/wrksheet.html

ACCRS: Algebra I.41-45

Algebra I.41: Represent data with plots on the real number line (dot plots, histograms, and box plots).

Algebra I.42: Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.

Algebra I.43: Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

Algebra I.44: Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.

Algebra I.45: Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.*

b. Informally assess the fit of a function by plotting and analyzing residuals. Fit a linear function for a scatter plot that suggests a linear association.

Mastered:

Students can represent data with plots on the real number line (dot plots, histograms, and box plots); use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. Students can summarize categorical data for two categories in two-way frequency tables, interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies) and recognize possible associations and trends in the data, and represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.*

b. Informally assess the fit of a function by plotting and analyzing residuals.

c. Fit a linear function for a scatter plot that suggests a linear association.

Present:

Students will create a PowerPoint project using the project planner which demonstrates understanding of, interpreting, and summarizing data. Histograms, dot plots and box plots will be discussed with reference to their origins, primary uses, and specific examples.

Going Forward:

Students will use some form of these graphic representations to help them better understand data when it is presented to them.

Present and Going Forward Vocabulary:

Dot plots, histograms, box plots, extremes, center

Career Connections:
Computer Information System Manager, Actuary, Computer Programmer

Advanced Understanding & Activity (Alternate activity): (Student page is located in Appendix A. The Project Planner is located in Appendix B.)

Project Planner

Student Instructions: Use the Project Planner to create a PowerPoint presentation or Prezi that includes:

- What histograms, dot plots, and box plots are and include demonstrations?
- Who is credited with their origins?
- When were these introduced?
- Description of the different parts of each.
- Why histograms, dot plots, and box plots are important.
- Collect data from your school to show as examples of each.
- Which data is best illustrated through each type of chart?
- Discuss the centtal tendencies (center attributes) of each, as well as the effect of outliers on each.

Literature Connections/Resources:
- Karl Pearson: http://en.wikipedia.org/wiki/Karl_Pearson
- John Tukey: http://en.wikipedia.org/wiki/John_Tukey

ACCRS: Algebra I.46
Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

Mastered:
Students can estimate the rate of change from a graph, and interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

Present:
Students will watch videos at http://tulyn.com/slope.htm and find slopes from ordered pairs, equations and a combination of the two.

Going Forward:
Students will notice the slopes they encounter every day and recognize that any incline or depression has a slope.

Present and Going Forward Vocabulary:
Estimate, rate of change, slope

Career Connections:
Human Resources Manager, Health Service Worker, Statistician

Advanced Understanding & Activity (Alternate activity): (Student page is located in Appendix A.)

The Slope Finder

Student Instructions: Watch the video “Slope Intercept Form” and “Finding the Slope Using Two Points” at http://tulyn.com/slope.htm. Solve the slope problems at http://math.about.com/od/allaboutslope/ss/Find-Slope-with-Graph-WP.htm, Find the slope given ordered pairs at http://www.kutasoftware.com/FreeWorksheets/Alg1Worksheets/Slope%20From%20Two%20Points.pdf Explain the importance of finding the slope of a line.

Literature Connections/Resources:
http://tulyn.com/slope.htm

ACCRS: Algebra I. 47-48

Algebra I. 47: Compute (using technology) and interpret the correlation coefficient of a linear fit.

Algebra I. 48: Distinguish between correlation and causation.

Mastered:	**Present:**	**Going Forward:**
Students can compute (using technology) and interpret the correlation coefficient of a linear fit, and distinguish between correlation and causation.	Students will take a survey, draw a scatter plot on Excel, find the correlation coefficient of the data and determine if there is a connection between correlation and causation. They will then use the Thinker Key Reverse.	Students will think of other relationships which affect school attendance and behavior, that could be related to sleep.

Present and Going Forward Vocabulary:

Correlation coefficient, causation

Career Connections:

Advertising, Medical Assistants, Computer Programmers.

Advanced Understanding & Activity (Alternate activity): (Student page is located in Appendix A.)

Solve the PUZZLE and CHALLENGE

Student Instructions:

1. Watch the video at http://www.youtube.com/watch?v=s2TVkYmmCAs. Take a survey in your math class asking each student the following questions:
   - How many hours of sleep do you get?
   - What time do you arrive at school?
2. Using Excel, create a scatter plot with your data to find the Corel correlation coefficient.
3. Once you have the correlation coefficient, speculate on the causation of not eating breakfast.
4. Read the article about causation at http://www.statistics-help-online.com/node50.html. How does that change your thinking about question #4?
5. Now brainstorm ten things that are not caused by lack of sleep.

Literature Connections/Resources:

- http://www.youtube.com/watch?v=s2TVkYmmCAs.
- http://www.statistics-help-online.com/node50.html.

ACCRS: Algebra I. 49-50

Algebra I.49: Describe events as subsets of a sample space (the set of outcomes), using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").

Algebra I.50: Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.

Mastered:	**Present:**	**Going Forward:**
Students can describe events as subsets of a sample space (the set of outcomes), using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not"), and understand that two events A and B are independent if the probability of A and B	Students will solve challenging puzzles dealing with probability and independent events.	Students will find activities in their lives at school and home in which knowing the probability would be of interest.

occurring together is the product of their probabilities, and use this characterization to determine if they are independent.		

Present and Going Forward Vocabulary:
Probability, independent events

Career Connections:
Computer Information System Manager, Actuary, Computer Programmer

Advanced Understanding & Activity (Alternate activity): (Student page is located in Appendix A.)

Take the Challenge!

Student Instructions: Solve the Puzzle and Challenge at http://www.mathgoodies.com/puzzles/probability_puzzles.html, then click on "Unit on Probability Theory," select and complete "Challenge Activities."

Literature Connections/Resources:
http://www.mathgoodies.com/puzzles/probability_puzzles.html