ACCRS: Algebra II.1-2

Algebra II.1: Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.

Algebra II.2: Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

Mastered:
Students know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.

Students can use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

Present:
Students will use complex number operations to determine if a given set of complex numbers are in the Mandelbrot set, and will graph those numbers on a complex plane.

Going Forward:
Students will find examples of fractals in nature, as well as simple mathematical fractals, including the Cantor set, Sierpinski Triangle, and Koch Snowflake.

Present and Going Forward Vocabulary:
Complex number, real part, imaginary part, complex plane, sequence, bounded, unbounded, fractal, Mandelbrot set

Career Connections:
Computer Graphic Designer, Cryptographer

Advanced Understanding & Activity (Alternate activity): (Student page is located in Appendix A.)

Mandelbrot or Julia Sets

Student Instructions: Students will complete the following items:

One way to tell if a complex number c is in the Mandelbrot set is to look for a repeating pattern of outputs when plugged into the formula $z^2 + c$, starting with $z = 0$ and plugging that output value back in for z, keeping c the same each time.

Look for a pattern in $z^2 + c$ for the following values of c: $i, 1, \frac{1}{4}$

If there is a pattern, each of those numbers in the pattern is in the Mandelbrot set. Graph these numbers on the complex plane. If the sequence does not form a pattern (is unbounded), say so, and leave these values un-graphed.

Once you have graphed those values that are in the Mandelbrot set, look up a graph of the entire Mandelbrot set online. You can create your own Mandelbrot set as you explore zoomed-in details of the set at the University of Utah. Explain what you discovered as you explored the set.

Literature Connections/Resources:

- Mandelbrot, Benoit. *The Fractal Geometry of Nature*. WH Freeman, 1982.
- http://en.wikipedia.org/wiki/Mandelbrot_set
- http://www.math.utah.edu/~pa/math/mandelbrot/mandelbrot.html

ACCRS: Algebra II.3-5

Algebra II.3: Solve quadratic equations with real coefficients that have complex solutions.

Algebra II.4: (+) Extend polynomial identities to the complex numbers.
Example: Rewrite $x^2 + 4$ as $(x + 2i)(x - 2i)$.

Algebra II.5: (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Mastered:
Students can solve quadratic equations with real coefficients that have complex solutions. Students can extend polynomial identities to the complex numbers.

Present:
Students will research the history of complex numbers, making note of the contributions of key figures including Cardano, Descartes, and Euler.

Going Forward:
Students will explore the trigonometric form of complex numbers, including the unique, simple properties of multiplication and division of complex numbers in that form.

Students know the Fundamental Theorem of Algebra; and can show that it is true for quadratic polynomials.		

Present and Going Forward Vocabulary:
Trigonometric form of complex numbers

Career Connections:
Environmental Engineer, Industrial Designer

Advanced Understanding & Activity (Alternate activity): (Student page is located in Appendix A.)

History of Complex Numbers

Student Instructions: Research the history of complex numbers, making note of the contributions of key figures including Cardano, Descartes, and Euler. Answer the following questions:

1. Who discovered complex numbers and in what context? (Note that several individuals throughout history independently discovered them).
2. What properties of real numbers fail when applied to the complex numbers and why?
3. What other ways are there to represent complex numbers? (besides a+bi)
4. What are some applications of complex numbers?
5. Why are complex numbers important?

Develop an infographic that includes your researched information. Explain how the contributions of key figures affected complex numbers and why these contributions are significant to our study of complex numbers today.

Literature Connections/Resources:
- http://en.wikipedia.org/wiki/Complex_number
- http://www.searchenginejournal.com/5-unbeatable-types-of-infographic-free-tools-to-create-them/27010/

ACCRS: Algebra II.6-7

Algebra II.6: Interpret expressions that represent a quantity in terms of its context.
a. Interpret parts of an expression such as terms, factors, and coefficients.
b. Interpret complicated expressions by viewing one or more of their parts as a single entity.
Example: Interpret $P(1+r)^n$ as the product of P and a factor not depending on P.

Algebra II.7: Use the structure of an expression to identify ways to rewrite it.
Example: See $x^4 – y^4$ as $(x^2)^2 – (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 – y^2)(x^2 + y^2)$.

Mastered:	**Present:**	**Going Forward:**
Students can interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. Students can use the structure of an expression to identify ways to rewrite it.	Students will solve problems involving volumes of hollow solids, where factors represent interior and exterior dimensions.	Students will determine interior and exterior surface area of the hollow solids given in the activity.

Present and Going Forward Vocabulary:
Volume, surface area

Career Connections:
Industrial Designer, Civil Engineer, Architect

Advanced Understanding & Activity (Alternate activity): (Student page is located in Appendix A.)

Hollow World

Student Instructions: A hollow cube has walls with a thickness of 1 cm. Complete the table below to compare the volume of the space inside the cube to the volume of the materials that make up the outer walls of the cube. Content in red would be what the students would supply. Note that the outside volume is computed by adding 2 because the thickness of the walls would add to all sides.

Inside Dimensions	Inside Volume	Outside Volume	Volume of materials	Comparison of inside volume to volume of materials
1 cm	1 cubic cm	27 cubic cm	26 cubic cm	1/26
1.5 cm	3.375 cubic cm	42.875 cubic cm	39.5 cubic cm	15.625/75.5
2 cm	8 cubic cm	64 cubic cm	56 cubic cm	8/56 or 1/7
2.5 cm	15.625 cubic cm	91.125 cubic cm	75.5 cubic cm	15.625/75.5
3 cm	27 cubic cm	125 cubic cm	98 cubic cm	27/98

Now consider a hollow cube with walls having a thickness of 1 cm. If the volume of the space inside this cube is one eighth the volume of the material that was used to construct the cube, determine the outer dimension of the cube to the nearest hundredth.

Student Instructions: The walls of a spherical shell have a thickness of 8mm. Complete the table below to compare the volume of the inside of the sphere to the volume of the materials that make up the walls of the sphere. Content in red would be what the students would supply. Note that the outside volume is computed by adding 2 because the thickness of the walls would add to all sides.

Inside Dimensions	Inside Volume	Outside Volume	Volume of materials	Comparison of inside volume to volume of materials
5 mm	(500/3)*pi cubic cm	(8788/3)*pi cubic cm	(8288/3)*pi cubic cm	125/2072
6 mm	288*pi cubic cm	(10976/3)*pi cubic cm	(10112/3)*pi cubic cm	864/10112
7 mm	(1372/3)*pi cubic cm	4500*pi cubic cm	(12128/3)*pi cubic cm	343/3032
8 mm	(2048/3)*pi cubic cm	(16384/3)*pi cubic cm	(14336/3)*pi cubic cm	1/7

A spherical shell has walls with a thickness of 8mm. The volume of the space inside the sphere is one-tenth the volume of the material that was used to construct the shell. Find the outer radius of the shell.

Students completing this activity might also be interested in learning about why boxes are designed differently for various products. The Web site http://en.wikipedia.org/wiki/Corrugated_box_design is easy to read and gives some valuable information about how design impacts cushioning, moving, stacking, etc.

Literature Connections/Resources:

- http://www.onlinemathlearning.com/volume-formula.html
- http://en.wikipedia.org/wiki/Corrugated_box_design

ACCRS: Algebra II.8

Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.
Example: Calculate mortgage payments.

Mastered:
Students can derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.

Present:
Students will solve problems involving infinite geometric series.

Going Forward:
Students will use geometric series to convert repeating decimals to fractional form, and to simplify continued fractions.

Present and Going Forward Vocabulary:
Infinite geometric series, continued fractions

Career Connections:
Ecologist, Physicist

Advanced Understanding & Activity (Alternate activity): (Student page is located in Appendix A.)

To Infinity and Beyond!

Student Instructions: When the common ratio r of a geometric series is such that $0 < |r| < 1$, the sum of the series is finite even if the series itself is infinite.

Draw a picture illustrating the sum of the infinite geometric series $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots$

Use the formula $S_\infty = \frac{a_1}{1-r}$ to compute the sum of the above series algebraically.

Complete the activities at the following link. You will derive the formula visually as you work through these real-world problems. You can explore the other real-world applications of a geometric series.
http://math.rice.edu/~lanius/Lessons/Series/infinite.htm

If you enjoy working with infinite geometric series, explore this activity page:
www.math.uakron.edu/amc/Discrete/InfiniteGeometricSeries_NSPIRE.doc

Literature Connections/Resources:
- http://www.mathwords.com/i/infinite_geometric_series.htm
- http://www.purplemath.com/modules/series5.htm
- http://www.emathzone.com/tutorials/algebra/infinite-geometric-series.html
- http://www.mathgym.com.au/htdocs/infin.htm
- http://math.rice.edu/~lanius/Lessons/Series/infinite.htm
- www.math.uakron.edu/amc/Discrete/InfiniteGeometricSeries_NSPIRE.doc

ACCRS: Algebra II.9-11

Algebra II.9: Understand that polynomials form a system analogous to the integers; namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Algebra II.10: Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x – a$ is $p(a)$, so $p(a) = 0$ if and only if $(x – a)$ is a factor of $p(x)$.

Algebra II.11: Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Mastered:
Students understand that polynomials form a system analogous to the integers; namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply

Present:
Students will use polynomial long division or synthetic division, Descartes' Rule of Signs, and the Rational Zero Test to factor and find the zeros of higher degree polynomials.

Going Forward:
Students will study the lead term test, and learn about the significance of the multiplicity of a particular zero of a polynomial function in order to graph the functions presented in this activity.

polynomials.
Students know and can apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$. Students can identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Present and Going Forward Vocabulary:
Polynomial long division, synthetic division, Descartes' Rule of Signs, Rational Zero Test, lead term test, multiplicity of a zero

Career Connections:
Accountant, Nurse, Banker, Architect, Physicist

Advanced Understanding & Activity (Alternate activity): (Student page is located in Appendix A.)

Polynomial Factoring

Students will choose one or more links from resources to explore Polynomial Factoring to learn how to factor higher degree polynomials based on the Rational Zero Test and synthetic division. Then, the student will factor and find all zeros of the polynomials on the worksheet available at http://edhelper.com/polynomials223.htm.

Literature Connections/Resources:

- Polynomial regression: http://www.enotes.com/topic/Polynomial_regression
- Polynomial Regression data fit applet: http://www.arachnoid.com/polysolve/index.html
- Sports data polynomial regression: http://www.sportsci.org/resource/stats/polynomial.html
- http://mathworld.wolfram.com/SyntheticDivision.html
- http://www.purplemath.com/modules/solvpoly.htm
- http://www.sosmath.com/algebra/factor/fac10/fac10.html
- http://www.sparknotes.com/math/algebra2/polynomials/problems_3.html
- http://www.purplemath.com/modules/drofsign.htm
- http://edhelper.com/polynomials223.htm

ACCRS: Algebra II.12-13

Algebra II.12: Prove polynomial identities and use them to describe numerical relationships. Example: The polynomial identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.

Algebra II.13: (+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined, for example, by Pascal's Triangle. (The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.)

Mastered:	Present:	Going Forward:
Students can prove polynomial identities and use them to describe numerical relationships.	Students will use the binomial theorem to show that the number of possible subsets of a group of objects is 2^n.	Students will research the special case of the Binomial Theorem that results in the Taylor Series, and determine

Students know and can apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined, for example, by Pascal's Triangle.		some of the many uses of this series, especially for approximating other functions.

Present and Going Forward Vocabulary:
Binomial theorem, factorial, combination "n choose k", Taylor series

Career Connections:
Cryptographer, Statistician, Economist, Electrical Engineer

Advanced Understanding & Activity (Alternate activity): (Student page is located in Appendix A.)

Binomial Theorem

Student Instructions: Using the binomial theorem, expand $(1+1)^n$. Observe that this expression is equivalent to that which represents the number of all possible subsets of a set with n elements: $\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \cdots + \binom{n}{n-1} + \binom{n}{n}$, where $\binom{n}{k} = \frac{n!}{k!(n-k)!}$ and $n! = n(n-1)(n-2)(n-3)\cdots 3\cdot 2\cdot 1$

Go to Khan Academy to view the video at http://www.khanacademy.org/math/precalculus/v/binomial-theorem--part-1. Then complete the activity at the end of the lesson.

Literature Connections/Resources:

- Khan Academy video tutorials on the Binomial theorem: http://www.khanacademy.org/math/precalculus/v/binomial-theorem--part-1
- http://www.math.hmc.edu/calculus/tutorials/binomial_thm/binomial_thm.pdf
- http://en.wikipedia.org/wiki/Taylor_series

ACCRS: Algebra II.14-15; Algebra II.25

Algebra II.14: Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or for the more complicated examples, a computer algebra system.

Algebra II.15: (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

Algebra II.25: Graph functions expressed symbolically, and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

a. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
b. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
c. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

Mastered:	Present:	Going Forward:
Students can rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$,	Students will learn how to graph rational functions by finding intercepts and asymptotes.	Students will research the relationship between asymptotes and limit values of a function.

using inspection, long division, or for the more complicated examples, a computer algebra system.
Students understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
Students can graph functions expressed symbolically, and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

Present and Going Forward Vocabulary:
Rational function, polynomial long division, intercept, asymptote, limit

Career Connections:
Engineer, Manufacturer, Medical Researcher, Doctor

Advanced Understanding & Activity (Alternate activity): (Student page is located in Appendix A.)

Asymptotes and Sketching Graphs

Student Instructions: Students will follow the tutorials provided on Purple Math (http://www.purplemath.com/modules/asymtote.htm) to learn how to find vertical, horizontal, and oblique asymptotes of a rational function. Work example problems available at:
http://hotmath.com/help/gt
http://hotmath.com/help/gt/genericalg2/section_7_1.html and
http://www.wtamu.edu/academic/anns/mps/math/mathlab/col_algebra/col_alg_tut40_ratgraph.htm to practice finding asymptotes and sketching graphs. Additional Web sites are available to provide additional learning.

Literature Connections/Resources:

- Khan Academy video lecture on asymptotes of rational functions: http://www.khanacademy.org/math/algebra/ck12-algebra-1/v/asymptotes-of-rational-functions
- http://www.purplemath.com/modules/asymtote.htm
- http://www.purplemath.com/modules/grphrtnl.htm
- http://www.wtamu.edu/academic/anns/mps/math/mathlab/col_algebra/col_alg_tut40_ratgraph.htm
- http://www.asms.net/brewer/precalgraphingsummary.pdf

ACCRS: Algebra II.16-18

Algebra II.16: Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

Algebra II.17: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

Algebra II.18: Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. Example: Represent inequalities describing nutritional and cost constraints on combinations of different foods.

Mastered:	Present:	Going Forward:
Students can create equations and inequalities in one variable	Students will solve linear systems of equations in two and	Students will research the following questions:

and use them to solve problems, create equations in two or more variables to represent relation-ships between quantities, graph equations on coordinate axes with labels and scales, represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.	three variables by reducing an augmented matrix into reduced row-echelon form.	What is the determinant of a matrix? What does that quantity tell us about the system of equations represented by the matrix?

Present and Going Forward Vocabulary:
Linear system of equations, matrix, augmented matrix, row-echelon form

Career Connections:
Economist, Nutritionist, Chemist

Advanced Understanding & Activity (Alternate activity): (Student page is located in Appendix A.)

Problem Solving using Matrices

Student Instructions: Following the steps outlined at http://www.asms.net/brewer/precal-matrixoperations.pdf, solve a collection of linear system of equations problems from your textbook using matrices instead of graphical or other algebraic methods. Solve at least two each of two- and three-variable systems.

1. What happens to the matrix when the system has no solution?
2. What happens to the matrix when the system has infinitely many solutions?

Literature Connections/Resources:
http://www.asms.net/brewer/precal-matrixoperations.pdf

ACCRS: Algebra II.19
Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. Example: Rearrange Ohm's law $V = IR$ to highlight resistance R.

Mastered:	Present:	Going Forward:
Students can rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.	Students will solve word problems which require rearranging literal equations to solve for a particular variable.	Students will examine economic data available from http://www.bea.gov/ to compare corporate profits to those of an individual investing in a compound interest account.

Present and Going Forward Vocabulary:
Literal equation

Career Connections:
Economist, Physicist, Chemist

Advanced Understanding & Activity (Alternate activity): (Student page is located in Appendix A.)

Interest in Investments

Student Instructions: Complete the activity found at http://www.curriki.org/xwiki/bin/download/Coll_tteegard/DowntoBusiness/EquationsandFormulasActivityKEY2-DowntoBusiness.zip/Equations%20and%20Formulas%20Activity%20-%20Down%20to%20Business.pdf. This activity examines investments and interest to see why it is important to be able to solve for a particular variable in an equation.

Literature Connections/Resources:

- http://www.mhhe.com/math/devmath/hutchison/student/olc/graphics/hutchison01eia_s/ch02/others/ch02-4.pdf
- http://www.purplemath.com/modules/solvelit.htm
- http://www.algebralab.org/lessons/lesson.aspx?file=Algebra_FunctionsRelationsLiteralEquations.xml
- http://www.curriki.org/xwiki/bin/download/Coll_tteegard/DowntoBusiness/EquationsandFormulasActivityKEY2-DowntoBusiness.zip/Equations%20and%20Formulas%20Activity%20-%20Down%20to%20Business.pdf

ACCRS: Algebra II.20
Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

Mastered:	Present:	Going Forward:
Students can solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.	Students will solve problems involving a circle inscribed in a triangle. What if the circle is circumscribed about a triangle? Look up the formula for the radius of such a circle, and try to solve similar problems as for the inscribed case. How is the formula for an inscribed circle derived from Heron's formula?	Students will research real-world problems that involve the use of Heron's formula.

Present and Going Forward Vocabulary:
Inscribed circle, equilateral triangle, circumscribed circle, Heron's formula

Career Connections:
Computer Graphic Designer, Physicist

Advanced Understanding & Activity (Alternate activity): (Student page is located in Appendix A.)
Student Instructions: Solve the following:

The radius r of a circle inscribed in a triangle with sides of length a, b, and c is given by $r = \sqrt{\frac{(s-a)(s-b)(s-c)}{s}}$ where $s = \frac{1}{2}(a + b + c)$.

a. Find the length of the radius of a circle inscribed in a triangle with sides of 5 inches, 6 inches, and 7 inches.
b. The radius of a circle inscribed in an equilateral triangle measures 2 inches. What is the length of each side of the equilateral triangle?

Literature Connections/Resources:

- http://en.wikipedia.org/wiki/Incircle_and_excircles_of_a_triangle
- http://www.mathsisfun.com/geometry/construct-triangleinscribe.html
- http://aleph0.clarku.edu/~djoyce/java/elements/bookIV/propIV4.html
- http://en.wikipedia.org/wiki/Heron's_formula
- http://www.t3ww.org/pdf/Heron.pdf

ACCRS: Algebra II.21
Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

Mastered:	Present:	Going Forward:

Students can explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.	Students will use the graphical method of solving simultaneous equations in order to solve nonlinear systems of equations.	Students will apply this concept to solve any number of word problems involving one or more equations. Students will compare literal equations with different values to see when results are equivalent.

Present and Going Forward Vocabulary:
Simultaneous equation, nonlinear system of equations, literal equation

Career Connections:
Economist, Physicist

Advanced Understanding & Activity (Alternate activity): (Student page is located in Appendix A.)

X Marks the Spot

Student Instructions: Any system of equations in which each equation can be rewritten to solve for the same variable can be solved graphically by looking for intersections of the graphs. Starting with the system of linear equations given by the application problems available at http://www.themathpage.com/alg/word-problems3.htm, use your knowledge of the intersections of graphs to solve the systems of equations given by exercises 1-12 of the handout available from http://sehomeclass.bellinghamschools.org/Teachers/Henoch/Alg%20II/systemofNonLEq72.pdf.

Extension: Name many, varied and unusual professions in which you could use this skill. The list has been started for you:

1. Economist
2. Physicist

Literature Connections/Resources:

- http://dl.uncw.edu/digilib/mathematics/algebra/mat111hb/Izs/gsolve/gsolve.html
- http://www.themathpage.com/alg/word-problems3.htm
- http://www.purplemath.com/modules/syseqgen.htm
- http://sehomeclass.bellinghamschools.org/Teachers/Henoch/Alg%20II/systemofNonLEq72.pdf
- http://www.khanacademy.org/video/non-linear-systems-of-equations-2?playlist=Algebra%20I%20Worked%20Examples

ACCRS: Algebra II.22-23

Algebra II.22: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*

Algebra II.23: Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
Example: If the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.

Mastered:	Present:	Going Forward:
Students can interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Students can relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.	Students will apply knowledge of domain and graphs of simple functions in order to graph piecewise functions.	Students will find at least three examples of real-world applications of piecewise functions, graph those situations, and write functions to describe the situations.

Present and Going Forward Vocabulary:
Piecewise function

Career Connections:
Economist, Biologist

Advanced Understanding & Activity (Alternate activity): (Student page is located in Appendix A.

Piecewise Exercises

Student Instructions

After reading about piecewise functions, student will work through the exercises from Part I of the handout available from: http://www.ciclt.net/ul/okresa/MATHEMATICS%20II%20Unit%205%20Step%20and%20Piecewise%20Functions.pdf.

Literature Connections/Resources:

- http://jwilson.coe.uga.edu/emt668/EMAT6680.Folders/Barron/unit/Lesson%204/4.html
- http://www.ciclt.net/ul/okresa/MATHEMATICS%20II%20Unit%205%20Step%20and%20Piecewise%20Functions.pdf
- http://www.youtube.com/watch?v=Bu9sMDHqnkc

ACCRS: Algebra II.24

Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Mastered:	Present:	Going Forward:
Students can calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval, and estimate the rate of change from a graph.	Students will use the difference quotient to estimate the instantaneous rate of change (derivative).	Students will apply knowledge of velocity to acceleration. OR: Students will create a simulation around one of the problems, then video or blog it to share with others.

Present and Going Forward Vocabulary:
Difference quotient

Career Connections:
Engineer, Biologist, Chemist, Physicist

Advanced Understanding & Activity (Alternate activity): (Student page is located in Appendix A.)

Feel the Need for Speed!

Student Instructions: Follow the examples and explanation provided in the links under resources, and then work exercises from the handout at http://math.ucsd.edu/~ashenk/Section2_2.pdf.

An object in free-fall is governed by the equation $s(t) = \frac{1}{2}gt^2 + v_0 t + s_0$, where

- s_0 is the initial height of the object
- v_0 is the initial velocity of the object (if dropped from a still position, $v_0 = 0$)
- g is the acceleration due to gravity $(-32\, {}^{ft}/_{s^2} \;\; or \;\; -9.8\, {}^{m}/_{s^2})$
- $s(t)$ is the functional notation indicating that position s is a function of the time t

The average rate of change of position, or average velocity, is given by the formula

$$v_{avg} = \frac{change\ in\ position}{change\ in\ time} = \frac{s(t_2) - s(t_1)}{t_2 - t_1}$$

In order to determine the instantaneous velocity, we look to the difference quotient of the function $s(t)$. In general, the difference quotient for a function $f(x)$ is given by $\frac{f(x+h)-f(x)}{h}$, where h is some small distance away from x. If we look at what happens as h gets really, really small, approaching 0, we achieve the instantaneous velocity.

OR: Choose one problem. Calculate the solution, then make up a scenario or realistic simulation to go with the problem. Remember to include professionals in your scenario who would naturally be included. You will write your scenario or simulation like a play, giving a setting, characters, a plot that includes a problem to be solved with the equation you chose, and a solution. You may include dialogue, if you choose.

Once your teacher approves your scenario or simulation, you may want to film or blog it.

Literature Connections/Resources:

- http://www.mathwords.com/d/difference_quotient.htm
- http://www.algebralab.org/studyaids/studyaid.aspx?file=Calculus_6-22.xml
- http://math.ucsd.edu/~ashenk/Section2_2.pdf

ACCRS: Algebra II.26-27

Algebra II.26: Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

Algebra II.27: Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
Example: Given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

Mastered:
Students can write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function, and compare properties of two functions each represented in a different way.

Present:
Students will explore the difference between a function given explicitly and implicitly.

Going Forward:
Students will examine real-world applications of conic sections, such as planetary orbits and "whispering rooms."

Present and Going Forward Vocabulary:
Implicit and explicit functions, conic sections

Career Connections:
Astronomer, Astrophysicist, Optical Engineer, Navigator, Architect, Acoustical Engineer

Advanced Understanding & Activity (Alternate activity): (Student page is located in Appendix A.)

Implicit to Explicit

Student Instructions: Examine a variety of implicitly-defined functions to determine which can easily be rewritten in an explicit form in order to make them easier to graph and evaluate. Start with the equation for a circle with center (0,0) and radius r: $x^2 + y^2 = r^2$. This equation itself is not that of a function, as clearly a circle fails the vertical line test. If we rearrange the formula to solve for y, however, we get two equations: one for the top half, and one for the bottom half of a circle: $y = \pm\sqrt{r^2 - x^2}$. What other implicitly defined functions can be easily rewritten, and what are some that can't be? Conic sections are a good place to start. Once you have gathered at least five different implicitly-written functions, see if you can rewrite them explicitly to solve for one of the variables.

Literature Connections/Resources:
- http://www.wmueller.com/precalculus/newfunc/4.html
- http://math.furman.edu/~dcs/courses/math10/lectures/l-31.pdf
- http://www.purplemath.com/modules/ellipse4.htm

ACCRS: Algebra II.28-29

Algebra II.28: Combine standard function types using arithmetic operations.
Example: Build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.

Algebra II.29: Identify the effect on the graph of replacing *f*(*x*) by *f*(*x*) + *k*, *k f*(*x*), *f*(*kx*), and *f*(*x* + *k*) for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

Mastered:

Students can combine standard function types using arithmetic operations.

Students can identify the effect on the graph of replacing *f*(*x*) by *f*(*x*) + *k*, *k f*(*x*), *f*(*kx*), and *f*(*x* + *k*) for specific values of *k*.

Present:

Students will apply knowledge of basic graphing transformations and algebraic combinations of functions to graph algebraic combinations of various functions.

Going Forward:

Students will explore the Wolfram Demonstrations Project linked to under "Resources" to see how graphs of compositions of functions are related to the functions that make them up.

Present and Going Forward Vocabulary:

Composition of functions

Career Connections:

Medical Field, Physicist, Biologist

Advanced Understanding & Activity (Alternate activity): (Student page is located in Appendix A.)

Combining Graphs of Functions

Student Instructions: Read the tutorial on combining graphs of functions available from http://vhcc2.vhcc.edu/pc1fall9/properties_of_functions/combining_graphs_of_functions.htm
Then work the example problems provided on that page.

Literature Connections/Resources:
- http://vhcc2.vhcc.edu/pc1fall9/properties_of_functions/combining_graphs_of_functions.htm
- http://demonstrations.wolfram.com/CompositionOfFunctions/

ACCRS: Algebra II.30

Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse, and write an expression for the inverse.

Example: $f(x) = 2x^3$ or $f(x) = (x+1)/(x-1)$ for $x \neq 1$.

Mastered:

Students can solve an equation of the form $f(x) = c$ for a simple function f that has an inverse, and write an expression for the inverse.

Present:

Students will demonstrate that formulas for converting between Celsius and Fahrenheit are inverses, and solve word problems using these formulas.

Going Forward:

Students will find other common functions that are inverses (for example logarithms and exponential functions), and investigate the relationships between the graphs of inverse functions.

Present and Going Forward Vocabulary:

Celsius, Fahrenheit

Career Connections:

Biologist, Chemist, Economist, Physicist, Medical Field

Advanced Understanding & Activity (Alternate activity): (Student page is located in Appendix A.)

Inverse Functions

Student Instructions: Celsius and Fahrenheit are related by the formula C = (5/9) × (F - 32). The formula for Fahrenheit is the inverse of this function. Find the inverse function, and then use the two formulas to determine the following temperatures in both Celsius and Fahrenheit:

1. Freezing point of water.
2. Average room temperature.
3. Average body temperature.
4. Boiling point of water.
5. Average temperatures of the surface of each of the planets in our solar system, as well as that of the sun.

Literature Connections/Resources:

- http://www.purplemath.com/modules/invrsfcn3.htm
- http://tutorial.math.lamar.edu/Classes/Alg/InverseFunctions.aspx
- http://www.wtamu.edu/academic/anns/mps/math/mathlab/col_algebra/ col_alg_tut32b_inverfun.htm
- http://water.me.vccs.edu/far2cel1.htm
- http://www.enchantedlearning.com/science/temperature/

ACCRS: Algebra II.31

For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers, and the base b is 2, 10, or e; evaluate the logarithm using technology.

Mastered:

Students can, for exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers, and the base b is 2, 10, or e; evaluate the logarithm using technology.

Present:

Students will apply knowledge of exponential and logarithmic functions to solve exponential growth and decay word problems.

Going Forward:

Students will use the logistic model for limited exponential growth to estimate the world's population.

Present and Going Forward Vocabulary:

Exponential growth, exponential decay, half-life, doubling time, logistic model

Career Connections:

Biologist, Chemist, Forensic Scientist

Advanced Understanding & Activity (Alternate activity): (Student page is located in Appendix A.)

Exponential Growth and Decay

Student Instructions: Learn about the basic exponential growth and decay formulas by watching videos on Khan Academy. Then study examples of these formulas at http://www.purplemath.com/modules/expoprob3.htm.

Solve problem numbers 13, 15, 17, 19, 21 at http://hotmath.com/help/gt/genericalg1/section_9_6.html. Then apply this knowledge to solve the problems available at http://www.algebralab.org/practice/practice.aspx?file=Word_LogarithmProblems.xml

Literature Connections/Resources:

- Khan Academy video on exponential growth: http://www.khanacademy.org/video/exponential-growth?playlist=Precalculus
- Khan Academy video on exponential decay
- http://www.purplemath.com/modules/expoprob3.htm
- http://hotmath.com/help/gt/genericalg1/section_9_6.html
- http://www.algebralab.org/practice/practice.aspx?file=Word_LogarithmProblems.xml

ACCRS: Algebra II.32-38

Algebra II.32: Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

Algebra II.33: Understand statistics as a process for making inferences about population parameters based on a random sample from that population.

Algebra II.34: Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation.
Example: A model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?

Algebra II.35: Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.

Algebra II.36: Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.

Algebra II.37: Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.

Algebra II.38: Evaluate reports based on data.

Mastered:

Students can:

- Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages; recognize that there are data sets for which such a procedure is not appropriate;
- Use calculators, spreadsheets, and tables to estimate areas under the normal curve;
- Understand statistics as a process for making inferences about population parameters based on a random sample from that population;
- Decide if a specified model is consistent with results from a

Present:

Students will complete a simple regression analysis of a set of data, and prepare a report on the significance of their findings.

Going Forward:

Students will conduct their own regression analysis and report using some of the data available from http://a2t910.blogspot.com/2010/04/regression-project.html.

given data-generating process, e.g., using simulation;
- Recognize the purposes of and differences among sample surveys, experiments, and observational studies;
- Explain how randomization relates to each;
- Use data from a sample survey to estimate a population mean or proportion;
- Develop a margin of error through the use of simulation models for random sampling;
- Use data from a randomized experiment to compare two treatments;
- Use simulations to decide if differences between parameters are significant;
- Evaluate reports based on data.

Present and Going Forward Vocabulary:
Regression analysis, correlation,

Career Connections:
Statistician, Actuary, Scientist, Engineer, Financial Advisor

Advanced Understanding & Activity (Alternate activity): (Student page is located in Appendix A.)

Regression Analysis

Student Instructions: Follow the tutorial outlined at http://sst-web.tees.ac.uk/external/U0000504/Notes/DataAnalysis/Regress/RegressTutorial.html.
Then they will complete several basic regression analysis exercises using either Microsoft Excel or a free trial of Minitab.

Literature Connections/Resources:
- Minitab Statistical Software (free 30-day trial): http://www.minitab.com/en-US/products/minitab/free-trial.aspx
- Regression Tutorial: http://easycalculation.com/statistics/learn-regression.php
- Correlation and Regression Tutorial: http://sst-web.tees.ac.uk/external/U0000504/Notes/DataAnalysis/Regress/RegressTutorial.html
- Guide to Regression in Minitab: http://www.stat.psu.edu/~lsimon/stat462/fa02/minitab/regression.htm
- Regression Project Data: http://a2t910.blogspot.com/2010/04/regression-project.html

ACCRS: Algebra II.39-40

Algebra II.39: (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).

Algebra II.40: (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

Mastered:	Present:	Going Forward:
Students can use probabilities to make fair decisions and analyze decisions and strategies using	Students will: predict and determine if games are fair, create a fair game from an unfair game,	Students will investigate the Monty Hall problem, at first by playing the interactive New

probability concepts.	organize and conduct a systematic investigation that includes empirical observations and theoretical analyses	York Times feature linked to under "Resources."

Present and Going Forward Vocabulary:
Probability, fair, random,

Career Connections:
Statistician, Economic Forecaster, Stockbroker, Marketing Manager

Advanced Understanding & Activity (Alternate activity): (Student page is located in Appendix A.)

What Are The Odds?

Student Instructions: Analyze the fairness of certain games by examining the probabilities of the outcomes. The explorations provide opportunities to predict results, play the games, and calculate probabilities. You should have had prior experiences with simple probability investigations, including flipping coins, drawing items from a set, and making tree diagrams. At the end of this activity, you will understand that the probability of an event is the ratio of the number of successful outcomes to the number of possible outcomes.

Complete the “Is It Fair Activity Sheet” and the “A Fair Hopper Activity Sheet” found at http://illuminations.nctm.org/LessonDetail.aspx?id=L290.

Choose a game you enjoy playing and determine if that game is fair by applying the process learned in this activity.

Literature Connections/Resources:

- http://illuminations.nctm.org/LessonDetail.aspx?id=L290
- http://www.nytimes.com/2008/04/08/science/08monty.html