

ADVANCED ALG/TRIG

Chapter 11 – Sequences and Series

Sequences

(an ordered list of numbers)

Arithmetic

D = common difference

Arithmetic mean =
sum of 2 numbers
divided by 2 (the
average)

Geometric

R = common ratio

Geometric
mean = square
root of product
of 2 numbers

Recursive formula

$$a_n = a_{n-1} + d; a_1 \text{ given}$$

Explicit formula

$$a_n = a_1 + (n-1)d$$

Recursive formula

$$a_n = a_{n-1} \cdot r; a_1 \text{ given}$$

Explicit formula

$$a_n = a_1 \cdot r^{(n-1)}$$

Series

(sum of terms in a sequence)

Arithmetic

Finite - ends
Infinite - does
not end...

Summation Notation
uses Sigma Σ ; has
lower and upper limits

Geometric

FINITE - ends; has
a sum

INFINITE
Converges when $|r| < 1$;
approaches a limit
Diverges when $|r| \geq 1$;
does not approach a limit

Sum of a Finite Arithmetic Series

$$S = n/2(a_1 + a_n)$$

Sum of a Finite Geometric Series

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

Sum of an Infinite Geometric Series

$$S_n = \frac{a_1}{1-r}$$

Hot Dog
Gist & Details
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Exponents

Gist

Zero Exponents

Details

$$n^0 = 1 \quad -n^0 = -1 \quad (-n)^0 = 1$$

Gist

Negative Exponents

Details

$$n^{-1} = 1/n \quad 1/n^{-1} = n$$

Gist

Multiplying Like Bases

Details

$$a^m \bullet a^n = a^{m+n}$$

Gist

Power to a Power

Details

$$(a^m)^n = a^{mn}$$

Gist

Quotient to a Power

Details

$$(a/b)^n = a^n / b^n$$

Gist

Product to a Power

Details

$$(ab)^n = a^n b^n$$

Gist

Dividing Like Bases

Details

$$\frac{a^m}{a^n} = a^{m-n}$$

Gist

Details

Transformations of Functions

Is about...

Transformations are useful in graphing any family of functions. If you can graph the functions, you can find possible maximums and minimums needed when maximizing or minimizing area, volume, etc.

Main Idea

Reflections

$$y = -f(x)$$

reflection across x

$$y = f(-x)$$

reflection across y

Main Idea

Translations

$$y = f(x) + k$$

up k units

$$y = f(x) - k$$

down k units

$$y = f(x + k)$$

right k units

$$y = f(x - k)$$

left k units

Main Idea

Dilations (Compressions and Stretches)

$$Y = a f(x)$$

If $a > 1$ or $a < -1$
vertical stretch

$$Y = f(ax)$$

If $a > 1$ or $a < -1$
horizontal compression

$$\text{If } -1 < a < 1$$

vertical compression

Polynomials

Algebraic expressions of problems when the task is to determine the value of one unknown number... X

Main idea

Monomials

Example:

$4x^3$ Cubic
5 Constant

Number of terms:

One = "mon"

$4x^3$

Degree:

Sum the exponents
of its variables

Main idea

Binomials

Example:

$7x + 4$ Linear
 $9x^4 + 11$ 4th degree

Number of terms:

Two = "bi"

$7x + 4$

Degree:

The degree of
monomial with
greatest degree

Main idea

Trinomials

Example:

$3x^2 + 2x + 1$
Quadratic

Number of terms:

Three = "tri"

$3x^2 + 2x + 1$

Degree:

The degree of the
monomial with
greatest degree

Main idea

Polynomials

Example:

$3x^5 + 2x^3 + 5x^2 + x - 4$
5th degree

Number of terms:

Many (> three)
="poly"

Degree:

The degree of the
monomial with
greatest degree

So what? What is important to understand about this?

The concept is important for the AHSGE Objective I-2. You may add or subtract polynomials when determining or representing a customer's order at a store.

Graphing a Quadratic Function by Hand

Main idea

Option 1

1. Complete the square in x to write the quadratic function in the form $f(x) = a(x - h)^2 + k$.
2. Graph the function in stages using transformations.

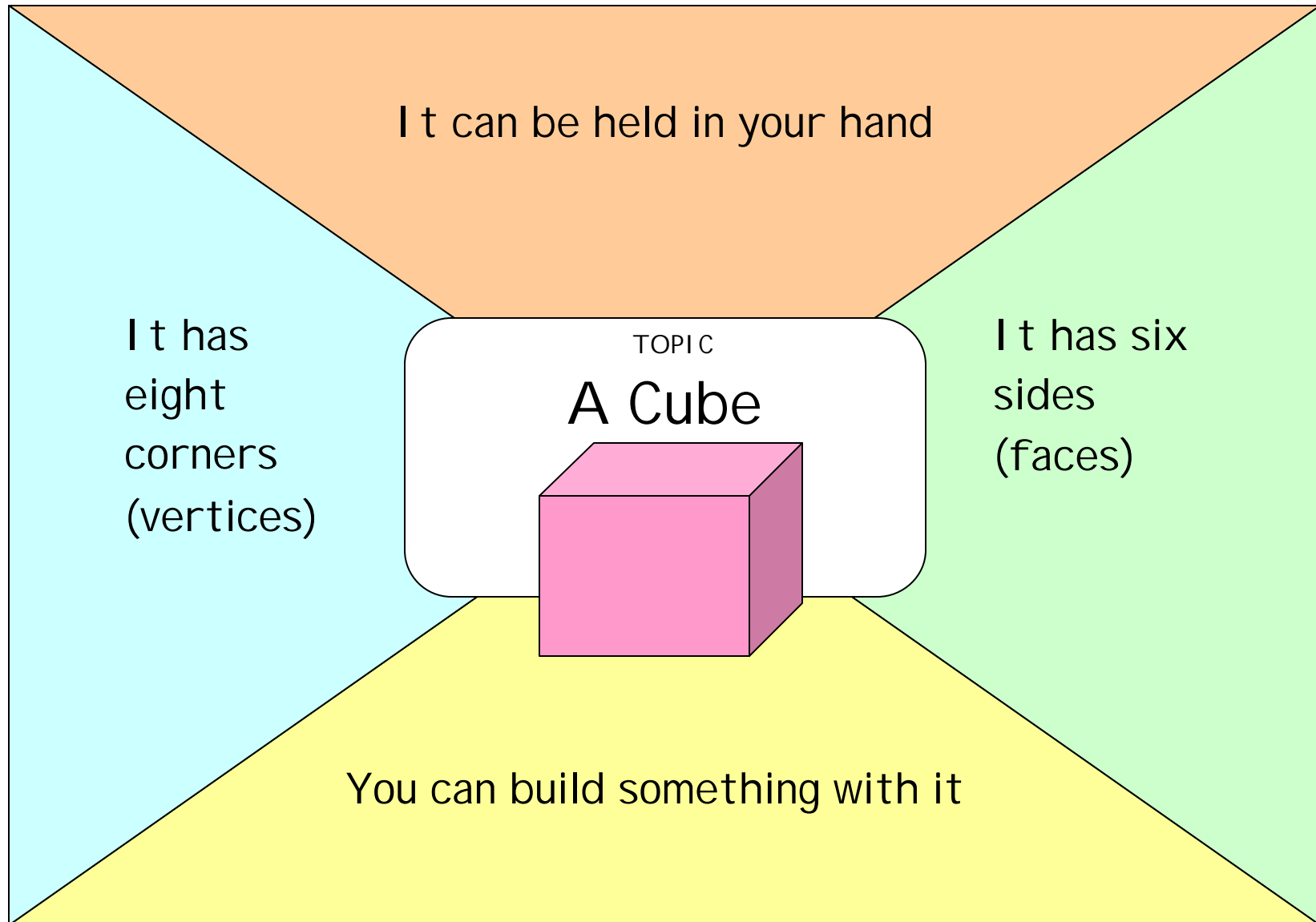
Main idea

Option 2

1. Determine the vertex $(-b/2a, f(-b/2a))$.
2. Determine the axis of symmetry, $x = -b/2a$
3. Determine the y-intercept, $f(0)$.
4. a) If the discriminant > 0 , then the graph of the function has two x-intercepts, which are found by solving the equation.
b) If the discriminant $= 0$, the vertex is the x-intercept.
c) If the discriminant < 0 , there are no x-intercepts.
5. Determine an additional point by using the y-intercept the axis of symmetry.
6. Plot the points and draw the graph.

So what? What is important to understand about this?

These options allow one to graph any quadratic function. A quadratic function models many of the physical, business, and area problems one see in real world situation. For example: maximize height of a projectile; maximize profit or revenue; minimize cost; Maximize/ minimize area or volume .



TOPIC

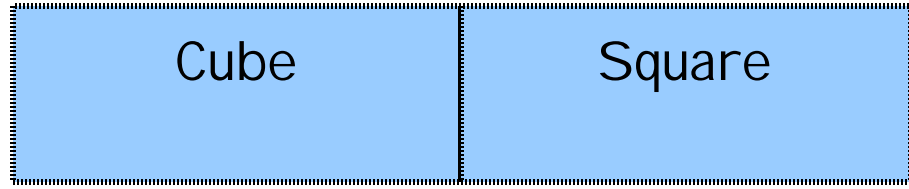
SQUARE

FLAT
(you cannot hold it in
your hand)

4 SIDES
(straight lines)

4 CORNERS
(vertices)

Can be used to
make a design
on paper



Main ideas	Cube Features	Square Features	
Sides	<i>Count the sides</i> A cube has _____ sides.	<i>Count the sides</i> A square has _____ sides.	Conclusion about this main idea The sides are shaped like squares
Corners	<i>Hold the block and count the corners</i> A cube has _____ corners.	<i>Touch each corner with your pencil</i> A square has _____ corners.	Conclusion about this main idea A cube has more corners
What can you do with it?	Build something	Make a design on paper	Conclusion about this main idea A square is easier to draw and use
	Conclusion about these features Cubes are Three Dimensional	Conclusion about these features Squares are One Dimensional	

Reducing Fractions

Is about ...

How to tell if a fraction is reduced to lowest terms

Is the numerator a "1"?

Yes, the fraction is reduced $1/6$

No, keep going $2/4 = 1/2$

Is the numerator 1 less than the denominator?

Yes, the fraction is reduced $5/6$

No, keep going $5/3 = 1 \frac{2}{3}$

Divide

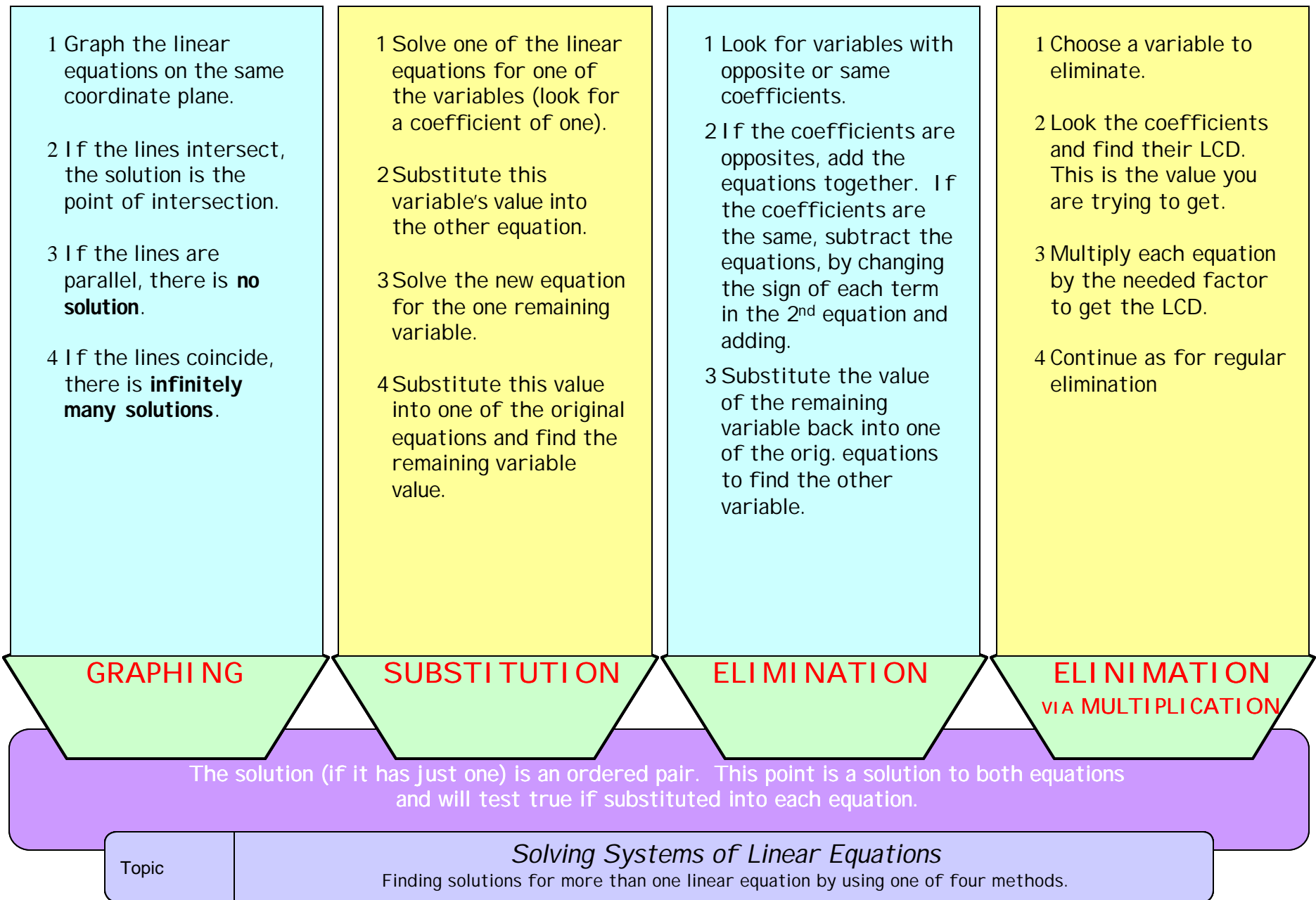
Divide both numbers and start again

$$\begin{array}{r} - \quad \quad \quad \underline{3} \quad \underline{3-3} = \underline{1} \\ \quad \quad \quad 6 \quad \underline{6-3} = \underline{2} \end{array}$$

Is there a number that will divide into both numbers?

No, the fraction is reduced $\frac{4}{7}$

Yes, keep going



Factor $Ax^2 + Bx + C$ Ratio Method

Name _____

Step 1

Factor out the GCF
 Write two binomials
 Signs: + C signs will be the same sign as the sign of b. - C negative and positive

Example: $24m^2 - 32m + 8$
 $8(3m^2 - 4m + 1)$
 $8(\quad) (\quad)$
 $8(\quad - \quad) (\quad - \quad)$

Step 2

Find AC
 $AC = 3$

Step 3

Find the factors of AC that will add or subtract (depends on the sign of c) to give u B.
 3 and 1 are the factors of AC that will add (note c is +) to give B.

Step 4

Write the ratio A/ Factor. Write the ratio Factor / C.
 Reduce.
 $3/1$ $1/1$ are reduced.

Step 5

Write the first ratio in the first binomial and the second ratio in the second binomial. $8(3x - 1) (x - 1)$
 Check using FOIL.

Five Steps

Why are these steps important?
 Following these steps will allow one to factor any polynomial that is not prime.
 Factoring a quadratic trinomial enable one to determine the x-intercepts of the parabola.
 Factoring also enables one to use the x-intercepts to

Solving Systems of Linear Equations

Is about ...

Finding solutions for more than one linear equation by using one of four methods.

Method 1

GRAPHING

1. Graph the linear equations on the same coordinate plane
2. If the lines intersect, the solution is the point of intersection.
3. If the lines are parallel, there is **no solution**.
4. If the lines coincide, there is **infinitely many solutions**.

Method 2

SUBSTITUTION

1. Solve one of the linear equations for one of the variables (look for a coefficient of one).
2. Substitute this variable's value into the other equation.
3. Solve the new equation for the one remaining variable.
4. Substitute this value into one of the original equations and find the remaining variable value.

Method 3

ELIMINATION

1. Look for variables with opposite or same coefficients.
2. If the coefficients are opposites, add the equations together. If the coefficients are the same, subtract the equations, by changing the sign of each term in the 2nd equation and adding.
3. Substitute the value of the remaining variable back into one of the orig. equations to find the other variable.

Method 4

ELIMINATION W/MULTIPLICATION

1. Choose a variable to eliminate.
2. Look the coefficients and find their LCD. This is the value you are trying to get.
3. Multiply each equation by the needed factor to get the LCD.
4. Continue as for regular elimination.

So what? What is important to understand about this?

The solution (if it has just one) is an ordered pair. This point is a solution to both equations and will test true if substituted into each equation.

SYSTEMS OF LINEAR INEQUALITIES

First Inequality

1. Graph: Solve for y and identify the slope and y -intercept. Graph the y -intercept on the y -axis and use the slope to locate another point. (Or, find the x and y intercepts and graph them on the x and y axis.)
2. Determine if the line is solid or dashed.
3. Pick a test point and test it in the original inequality. (If true, shade where the point is. If false, shade on the opposite side of the line.)

Second Inequality

1. Graph: Solve for y and identify the slope and y -intercept. Graph the y -intercept on the y -axis and use the slope to locate another point. (Or, find the x and y intercepts and graph them on the x and y axis.)
2. Determine if the line is solid or dashed.
3. Pick a test point and test it in the original inequality. (If true, shade where the point is. If false, shade on the opposite side of the line.)

Solution

1. Darken the area where the shaded regions overlap.
2. If the regions do not overlap, there is no solution.

Check

1. Choose a point in the darkened area.
2. Test in both original inequalities.
3. Correct if **both** test true.

Transformations of Functions

Main Idea

Reflections

Details

$y = -f(x)$ reflection across x

$y = f(-x)$ reflection across y

Main Idea

Translations

Details

$y = f(x) + k$ up k units

$y = f(x) - k$ down k units

$y = f(x + k)$ right k units

$y = f(x - k)$ left k units

Main Idea

Dilations (Compressions and Stretches)

Details

$Y = a f(x)$
If $a > 1$ or $a < -1$ vertical stretch
If $-1 < a < 1$ vertical compression

$Y = f(ax)$
If $a > 1$ or $a < -1$ horizontal compression
If $-1 < a < 1$ vertical compression

When will I ever use transformations

Transformations are useful in graphing any family of functions. If you can graph the functions, you can find possible maximums and minimums needed when maximizing or minimizing area, volume, etc. . .

Factor $Ax^2 + Bx + C$ **Trial and Error.**

Name _____

Five Steps

Step 1

Factor out the GCF. Example: $24m^2 - 32m + 8$
 $8(3m^2 - 4m + 1)$

Step 2

Write two binomials
 Signs: + c signs will be the same sign as the sign of b
 - c one negative and one positive. $8(\quad) (\quad)$

Step 3

List the factors of A and the Factors of B.
 $\frac{A = 3}{1, 3}$ $\frac{B = 1}{1, 1}$

Step 4

If C is positive, determine the factor combination of A and B that will **add** to give B.
 If C is negative, determine the factor combination of A and B that will **subtract** to give B.
 Since C is positive add to get B : $8(3x - 1)(x - 1)$

Step 5

Check using FOIL. $8(3x^2 - 3x - x + 1)$
 $= 8(3x^2 - 4x + 1)$
 $= 24x^2 - 32x + 8$

Why are these steps important?

Following these steps will allow one to factor any polynomial that is not prime.

Factoring a quadratic trinomial enables one to determine the x-intercepts of the parabola.

Factoring also enables one to use the x-intercepts to graph.

Factoring Polynomials Completely

Name _____

Step 1

Factor out the GCF

Example: $2x^3 - 6x^2 = 2x^2(x - 3)$

Always make sure the remaining polynomial(s) are factored.

Step 2

Two Terms: Check for "DOTS" (Difference of Two Squares)

1. Two terms
2. Difference
3. A and C are perfect squares

Example: $x^2 - 4 = (x - 2)(x + 2)$ see if the binomials will factor again. Check Using FOIL

Step 3

Three Terms: Check for "PST"

1. Check for "PST" $m^2 + 2mn + n^2$ or $m^2 - 2mn + n^2$. Factor using short cut
2. No "PST", Factor Using Ratio or Trial and error method
3. Check Using FOIL

Example PST: $c^2 + 10c + 25 = (c + 5)^2$ For examples of # 2, See Ratio-Trial/Error Method GC*

Step 4

Four Terms: Grouping

1. Group two terms together that have a GCF.
2. Factor out the GCF from each pair. Look for common binomial.
3. Re-write with common binomial times other factors in a binomial.

Example: $5t^4 + 20t^3 + 6t + 24 = (t + 4)(5t^3 + 6)$

Step 5

Make Sure that each Polynomial is factored completely.

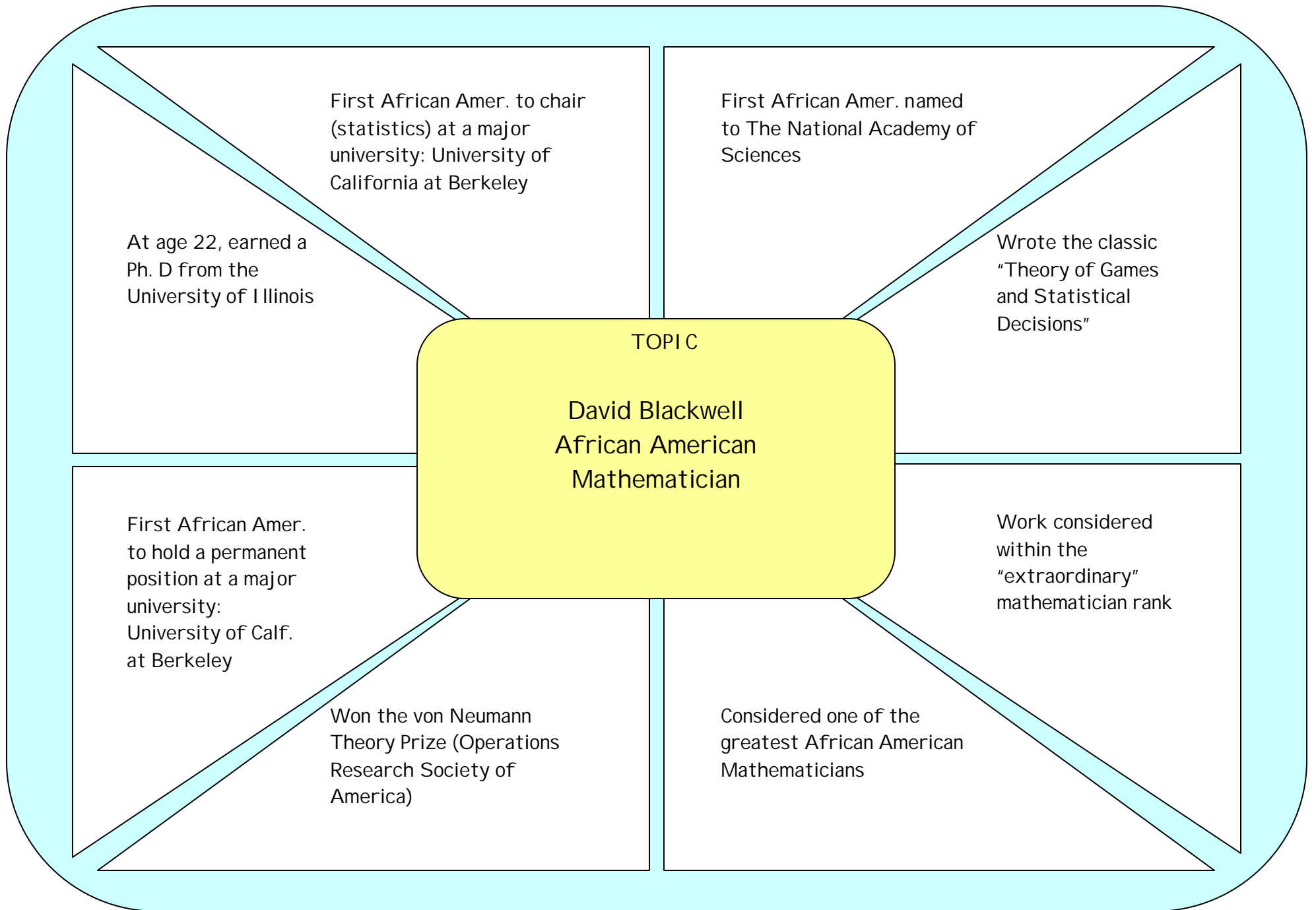
If you have tried steps 1 - 4 and the polynomial cannot be factored, the polynomial is Prime.

Five Steps

Why are these steps important?

Following these steps will allow one to factor any polynomial that is not prime.

Factoring allows one to find the x-intercepts and in turn graph the polynomial.



Topic

Divisibility

Details

Dividing numbers that don't have remainders

Main Idea

Two (2)

Details

If a number ends with
0, 2, 4, 6, 8

Main Idea

Five (5)

Details

If a number ends with
0 or 5

Main Idea

Ten (10)

Details

If a number ends with
0

Main Idea

Three (3)

Details

If the sum of the digits
can be divided by 3

Main Idea

Nine (9)

Details

If the sum of the digits
can be divided by 9

Polynomial Products

Multiplying Polynomials and Special Products

Main idea

Polynomials times
Polynomials

Monomial times a
Polynomial:
Use the distributive
property (x coeff +
exponents)
Example:
 $-4y^2 (5y^4 - 3y^2 + 2) = -$
 $20y^6 + 12y^4 - 8y^2$
Polynomial times a
Polynomial:
Use the distributive
Property
Example:
 $(2x - 3) (4x^2 + x - 6) =$
 $8x^3 - 10x^2 - 15x + 18$

Main idea

Binomials times
Binomials

FOIL:
F = **First**
O = **Outer**
I = **Inner**
L = **Last**

Example:
 $(3x - 5)(2x + 7) =$
 $6x^2 + 11x - 35$

Main idea

Square of a
Binomial

$(a + b)^2 =$
 $a^2 + 2ab + b^2$

 $(a - b)^2 =$
 $a^2 - 2ab + b^2$
1st term: Square the
first term
2nd term: Multiply two
terms and Double
3rd term: Square the
last term
Examples:
 $(x + 6)^2 =$
 $x^2 + 12x + 36$
 $(7m - 2p)^2 =$
 $49m^2 - 14mp + 4p^2$

Main idea

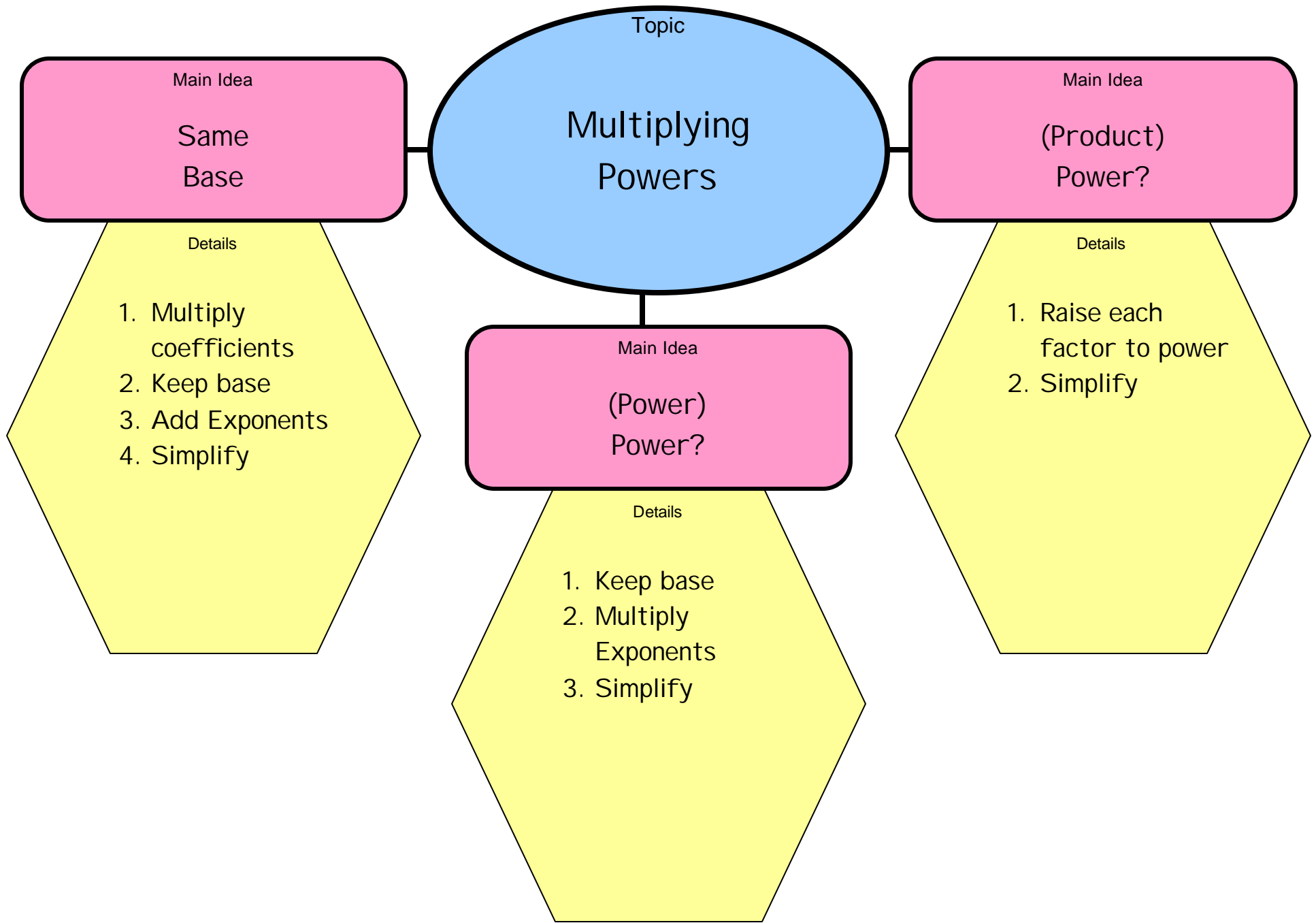
Difference of Two
Squares

DOTS:
 $(a + b) (a - b) =$
 $a^2 - b^2$

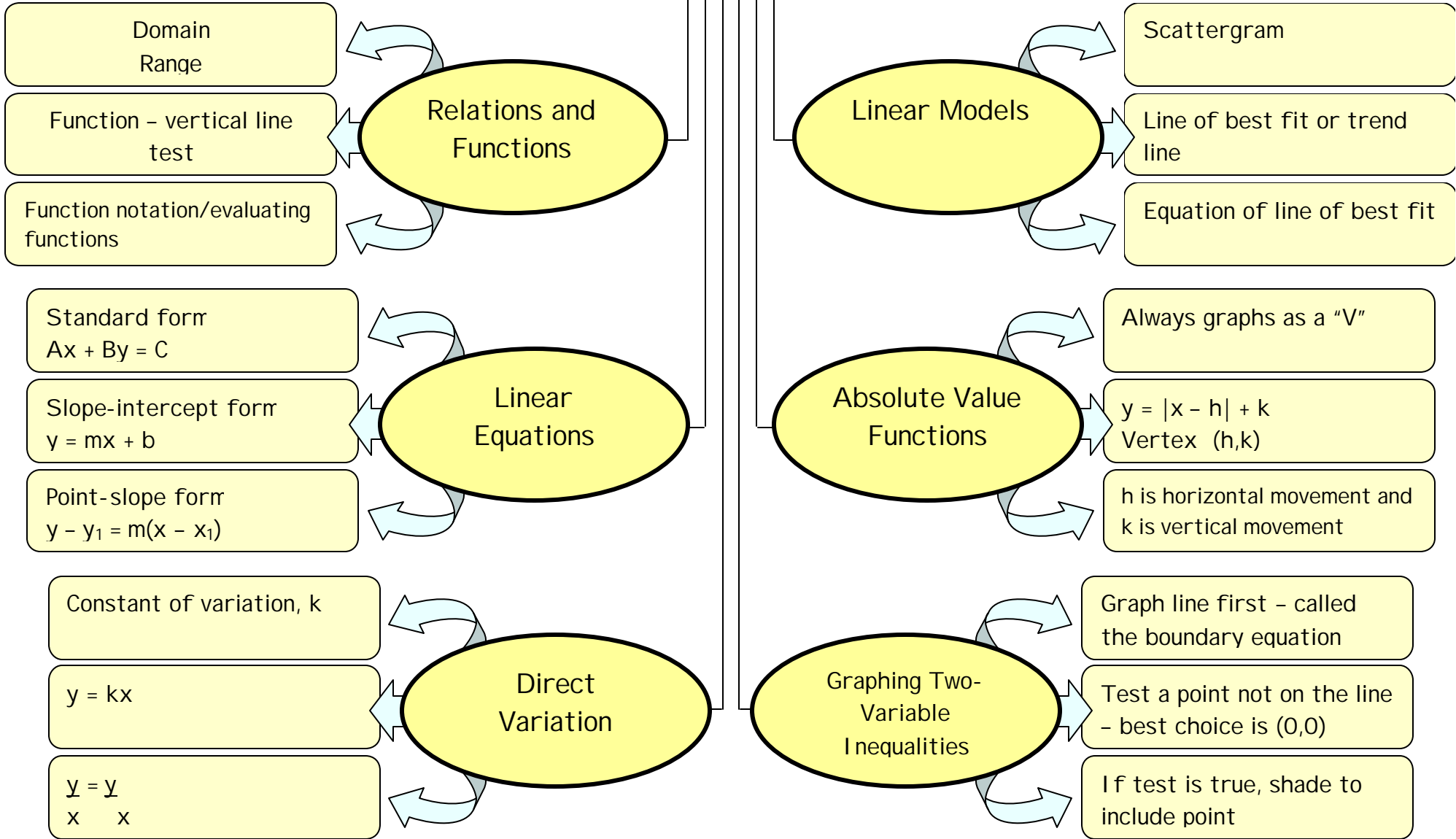
Example:
 $(t^3 - 6) (t^3 + 6) =$
 $t^6 - 36$

So what? What is important to understand about this?

AHSGE Objective: I -3. Applications include finding area and volume. Special products are used in graphing functions by hand. Punnett Squares in Biology.



Functions, Equations, and Graphs



Domain
Range

Function - vertical line test

Function notation/evaluating functions

Relations and Functions

Linear Models

Scattergram

Line of best fit or trend line

Equation of line of best fit

Standard form
 $Ax + By = C$

Slope-intercept form
 $y = mx + b$

Point-slope form
 $y - y_1 = m(x - x_1)$

Linear Equations

Absolute Value Functions

Always graphs as a "V"

$y = |x - h| + k$
Vertex (h,k)

h is horizontal movement and k is vertical movement

Constant of variation, k

$y = kx$

$\frac{y}{x} = \frac{y}{x}$

Direct Variation

Graphing Two-Variable Inequalities

Graph line first - called the boundary equation

Test a point not on the line - best choice is (0,0)

If test is true, shade to include point

Factor $A^2 - C^2$ **These are the steps to ...**
"DOTS" = Difference of Two Squares

Name _____

Step 1

Factor out the GCF. Example: $3x^4 - 48$
 $3(x^4 - 16)$

Step 2

Check for : 1.) 2 terms
 2.) Minus Sign
 3.) A and C are perfect squares

Step 3

Write two binomials
 Signs: One + ; One - $3(+) (-)$

Step 4

Write the numbers and variables before they were squared in the binomials. (Note: Any even power on a variable is a perfect square. . . just half the exponent when factoring) $x^2 + 4$) ($x^2 - 4$) $3($

Step 5

Check for DOTS in DOTS $3(x^2 + 4) (x^2 - 4)$
 $3(x^2 + 4) (x + 2)(x - 2)$
 Check using FOIL.

Five Steps

Why are these steps important? .
 Following these steps will allow one to factor any polynomial that is not prime.

 Factoring a quadratic trinomial enables one to determine the x-intercepts of a parabola.

 Factoring also enables one to use the x-intercepts to graph.

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Key Topic
Lines

is about...

2 kinds of lines

Main idea

Perpendicular

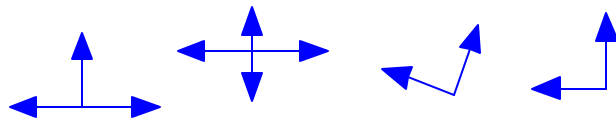
Detail

Essential because...

Intersect / meet

form right angles

forms a square corner



Main idea

Parallel

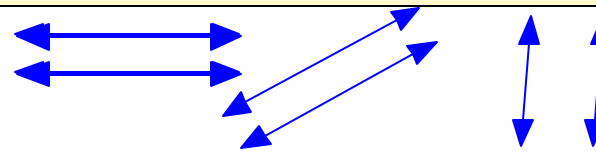
Detail

Essential because...

never intersect, meet, or touch

same plane

go in same direction



So what? What is important to understand about this?

There are only two types of lines

Key Topic
Angles

is about...

4 kinds of angles

Straight



180°

does not bend

Right



always 90°

square corners

perpendicular

Acute



less than 90°

1° -- 89°

small angle

Obtuse



greater than 90°

91° - 179°

wide angle

angle larger than right angle

So what? What is important to understand about this?

type of angle is determined by the degree of arch

