ADVANCED ALG/TRIG

Chapter 11 – Sequences and Series

Sequences

(an ordered list of numbers)

Arithmetic

D = common difference

Arithmetic mean = sum of 2 numbers divided by 2 (the average)

Recursive formula

$a_n = a_{n-1} + d$; a_1 given

Explicit formula

$a_n = a_1 + (n-1)d$

Geometric

R = common ratio

Geometric mean = square root of product of 2 numbers

Recursive formula

$a_n = a_{n-1} \bullet r$; a_1 given

Explicit formula

$a_n = a_1 \bullet r^{(n-1)}$

Series

(sum of terms in a sequence)

Arithmetic

Finite – ends

Infinite – does not end…

Summation Notation uses Sigma Σ; has lower and upper limits

Sum of a Finite Arithmetic Series

$S = n/2(a_1 + a_n)$

Geometric

FINITE – ends; has a sum

INFINITE

Converges when $|r|<1$; approaches a limit

Diverges when $|r| \geq 1$; does not approach a limit

Sum of a Finite Geometric Series

$S_n = \frac{a_1(1-r^n)}{1 - r}$

Sum of an Infinite Geometric Series

$S_n = \frac{a_1}{1 - r}$

Hot Dog
Gist & Details

Exponents

Gist
Zero Exponents
Details
$n^0 = 1$ $-n^0 = -1$ $(-n)^0 = 1$

Gist
Negative Exponents
Details
$n^{-1} = 1/n$ $1/n^{-1} = n$

Gist
Multiplying Like Bases
Details
$a^m \bullet a^n = a^{m+n}$

Gist
Power to a Power
Details
$(am)^n = a^{mn}$

Gist
Quotient to a Power
Details
$(a/b)^n = a^n / b^n$

Gist
Product to a Power
Details
$(ab)^n = a^n b^n$

Gist
Dividing Like Bases
Details
$$\frac{a^m}{a^n} = a^{m-n}$$

Gist

Details

Transformations of Functions

Is about…

Transformations are useful in graphing any family of functions. If you can graph the functions, you can find possible maximums and minimums needed when maximizing or minimizing area, volume, etc. . .

Main Idea: Reflections

- $y = -f(x)$ — reflection across x
- $y = f(-x)$ — reflection across y

Main Idea: Translations

- $y = f(x) + k$ — up k units
- $y = f(x) - k$ — down k units
- $y = f(x + k)$ — right k units
- $y = f(x - k)$ — left k units

Main Idea: Dilations (Compressions and Stretches)

- $Y = a\, f(x)$ — If $a > 1$ or $a < -1$ vertical stretch
- $Y = f(ax)$ — If $a > 1$ or $a < -1$ horizontal compression
- If $-1 < a < 1$ — vertical compression

Polynomials

Algebraic expressions of problems when the task is to determine the value of one unknown number… X

Main idea	Main idea	Main idea	Main idea
Monomials	**Binomials**	**Trinomials**	**Polynomials**
Example: $4x^3$ Cubic 5 Constant	Example: $7x + 4$ Linear $9x^4 + 11$ **4th degree**	Example: $3x^2 + 2x + 1$ Quadratic	Example: **$3x^5 + 2x^3 + 5x^2 + x - 4$** 5th degree
Number of terms: One = "mon" $4x^3$	Number of terms: Two = "bi" $7x + 4$	Number of terms: Three ="tri" $3x^2 + 2x + 1$	Number of terms: Many (> three) ="poly"
Degree: Sum the exponents of its variables	Degree: The degree of monomial with greatest degree	Degree: The degree of the monomial with greatest degree	Degree: The degree of the monomial with greatest degree

So what? What is important to understand about this?

The concept is important for the AHSGE Objective I-2. You may add or subtract polynomials when determining or representing a customer's order at a store.

Graphing a Quadratic Function by Hand

Main idea

Option 1

1. Complete the square in x to write the quadratic function in the form $f(x) = a(x - h)^2 + k$.
2. Graph the function in stages using transformations.

Main idea

Option 2

1. Determine the vertex (-b/2a, f(-b/2a)).
2. Determine the axis of symmetry, x = _b/2a
3. Determine the y-intercept, f(0).
4. a) If the discriminant > 0, then the graph of the function has two x-intercepts, which are found by solving the equation.
   b) If the discriminant = 0, the vertex is the x-intercept.
   c) If the discriminant < 0, there are no x-intercepts.
5. Determine an additional point by using the y-intercept the axis of symmetry.
6. Plot the points and draw the graph.

So what? What is important to understand about this?

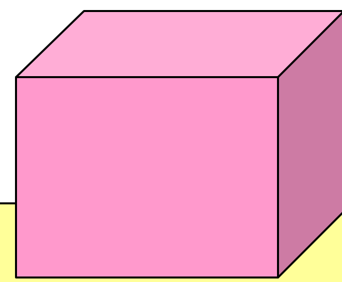
These options allow one to graph any quadratic function. A quadratic function models many of the physical, business, and area problems one see in real world situation. For example: maximize height of a projectile; maximize profit or revenue; minimize cost; Maximize/ minimize area or volume . . .

It can be held in your hand

It has eight corners (vertices)

TOPIC

A Cube



It has six sides (faces)

You can build something with it

TOPIC

SQUARE

FLAT
(you cannot hold it in your hand)

4 SIDES
(straight lines)

4 CORNERS
(vertices)

Can be used to make a design on paper

Main ideas	Cube Features	Square Features	
Sides	*Count the sides* A cube has _______ sides.	*Count the sides* A square has _______ sides.	Conclusion about this main idea The sides are shaped like squares
Corners	*Hold the block and count the corners* A cube has _______ corners.	*Touch each corner with your pencil* A square has ______ corners.	Conclusion about this main idea A cube has more corners
What can you do with it?	Build something	Make a design on paper	Conclusion about this main idea A square is easier to draw and use
	Conclusion about these features Cubes are Three Dimensional	Conclusion about these features Squares are One Dimensional	

Reducing Fractions

Is about …

How to tell if a fraction is reduced to lowest terms

Is the numerator a "1"?

Yes, the fraction is reduced 1/6

No, keep going 2/4 = 1/2

Is the numerator 1 less than the denominator?

Yes, the fraction is reduced 5/6

No, keep going 5/3 = 1 2/3

Is there a number that will divide into both numbers?

No, the fraction is reduced $\frac{4}{7}$

Yes, keep going

Divide

Divide both numbers and start again

\- $\frac{3}{6}$ $\frac{3-3}{6-3} = \frac{1}{2}$

Topic

Solving Systems of Linear Equations

Finding solutions for more than one linear equation by using one of four methods.

The solution (if it has just one) is an ordered pair. This point is a solution to both equations and will test true if substituted into each equation.

GRAPHING

1. Graph the linear equations on the same coordinate plane.
2. If the lines intersect, the solution is the point of intersection.
3. If the lines are parallel, there is **no solution**.
4. If the lines coincide, there is **infinitely many solutions**.

SUBSTITUTION

1. Solve one of the linear equations for one of the variables (look for a coefficient of one).
2. Substitute this variable's value into the other equation.
3. Solve the new equation for the one remaining variable.
4. Substitute this value into one of the original equations and find the remaining variable value.

ELIMINATION

1. Look for variables with opposite or same coefficients.
2. If the coefficients are opposites, add the equations together. If the coefficients are the same, subtract the equations, by changing the sign of each term in the 2nd equation and adding.
3. Substitute the value of the remaining variable back into one of the orig. equations to find the other variable.

ELINIMATION VIA MULTIPLICATION

1. Choose a variable to eliminate.
2. Look the coefficients and find their LCD. This is the value you are trying to get.
3. Multiply each equation by the needed factor to get the LCD.
4. Continue as for regular elimination

F actor **Ax^2 + Bx + C** Ratio Method

Name

Five Steps

Step 1

Example: $24m^2 - 32m + 8$

Factor out the GCF $8(3m^2 - 4m + 1)$

Write two binomials 8 ( ) ( )

Signs: + C signs will be the same sign as the sign of b . - C negative and positive

8(-) (-)

Step 2

Find AC

AC = 3

Step 3

Find the factors of AC that will add or subtract (depends on the sign of c) to give u B.

3 and 1 are the factors of AC that will **add** **(note c is +)** to give B.

Step 4

Write the ratio A/ Factor. Write the ratio Factor / C. Reduce.

3/1 **1/1** are reduced.

Step 5

Write the first ratio in the first binomial and the second ratio in the second binomial. 8(3x – 1) (x – 1)

Check using FOIL.

Why are these steps important? .

Following these steps will allow one to factor any polynomial that is not prime.

Factoring a quadratic trinomial enable one to determine the x-intercepts of the parabola.

Factoring also enables one to use the x-intercepts to

Solving Systems of Linear Equations

Is about …

Finding solutions for more than one linear equation by using one of four methods.

Method 1: GRAPHING

1. Graph the linear equations on the same coordinate plane
2. If the lines intersect, the solution is the point of intersection.
3. If the lines are parallel, there is **no solution**.
4. If the lines coincide, there is **infinitely many solutions**.

Method 2: SUBSTITUTION

1. Solve one of the linear equations for one of the variables (look for a coefficient of one).
2. Substitute this variable's value into the other equation.
3. Solve the new equation for the one remaining variable.
4. Substitute this value into one of the original equations and find the remaining variable value.

Method 3: ELIMINATION

1. Look for variables with opposite or same coefficients.
2. If the coefficients are opposites, add the equations together. If the coefficients are the same, subtract the equations, by changing the sign of each term in the 2^{nd} equation and adding.
3. Substitute the value of the remaining variable back into one of the orig. equations to find the other variable.

Method 4: ELIMINATION W/MULTIPLICATION

1. Choose a variable to eliminate.
2. Look the coefficients and find their LCD. This is the value you are trying to get.
3. Multiply each equation by the needed factor to get the LCD.
4. Continue as for regular elimination.

So what? What is important to understand about this?

The solution (if it has just one) is an ordered pair. This point is a solution to both equations and will test true if substituted into each equation.

SYSTEMS OF LINEAR INEQUALITIES

First Inequality

1. Graph: Solve for y and identify the slope and y-intercept. Graph the y-intercept on the y-axis and use the slope to locate another point. (Or, find the x and y intercepts and graph them on the x and y axis.)
2. Determine if the line is solid or dashed.
3. Pick a test point and test it in the original inequality. (If true, shade where the point is. If false, shade on the opposite side of the line.)

Second Inequality

1. Graph: Solve for y and identify the slope and y-intercept. Graph the y-intercept on the y-axis and use the slope to locate another point. (Or, find the x and y intercepts and graph them on the x and y axis.)
2. Determine if the line is solid or dashed.
3. Pick a test point and test it in the original inequality. (If true, shade where the point is. If false, shade on the opposite side of the line.)

Solution

1. Darken the area where the shaded regions overlap.
2. If the regions do not overlap, there is no solution.

Check

1. Choose a point in the darkened area.
2. Test in both original inequalities.
3. Correct if **both** test true.

Transformations of Functions

Main Idea

Reflections

Details

y = -f(x) reflection across x

y = f(-x) reflection across y

Main Idea

Translations

Details

y = f(x) + k up k units

y = f(x) – k down k units

y = f(x + k) right k units

y = f(x – k) left k units

Main Idea

Dilations (Compressions and Stretches)

Details

Y= a f(x)

If a > 1 or a < -1 vertical stretch

If –1 < a < 1 vertical compression

Y = f(ax)

If a > 1 or a < -1 horizontal compression

If –1< a < 1 vertical compression

When will I ever use transformations

Transformations are useful in graphing any family of functions. If you can graph the functions, you can find possible maximums and minimums needed when maximizing or minimizing area, volume, etc. . .

Factor $Ax^2 + Bx + C$ Trial and Error.

Name

Five Steps

Step 1

Factor out the GCF.

Example: $24m^2 - 32m + 8$

$8(3m^2 - 4m + 1)$

Step 2

Write two binomials

Signs: + c signs will be the same sign as the sign of b

- c one negative and one positive.

8(-) (-)

Step 3

List the factors of A and the Factors of B.

A = 3	B = 1
1, 3	1, 1

Step 4

If C is positive, determine the factor combination of A and B that will **add** to give B.

If C is negative, determine the factor combination of A and B that will **subtract** to give B.

Since C is positive add to get B : $8(3x - 1)(x - 1)$

Step 5

Check using FOIL.

$8(3x^2 - 3x - x + 1)$

$= 8(3x^2 - 4x + 1)$

$= 24x^2 - 32x + 8$

Why are these steps important?

Following these steps will allow one to factor any polynomial that is not prime.

Factoring a quadratic trinomial enables one to determine the x-intercepts of the parabola.

Factoring also enables one to use the x-intercepts to graph.

Factoring Polynomials Completely

Name

Five Steps

Step 1

Factor out the GCF

Example: $2x^3 - 6x^2 = 2x^2 (x - 3)$

Always make sure the remaining polynomial(s) are factored.

Step 2

Two Terms : Check for "DOTS" (Difference of Two Squares)

1. Two terms
2. Difference
3. A and C are perfect squares

Example: $x^2 - 4 = (x - 2)(x + 2)$ see if the binomials will factor again. Check Using FOIL

Step 3

Three Terms: Check for "PST"

1. Check for "PST" $m^2 + 2mn + n^2$ or $m^2 - 2mn + n^2$. Factor using short cut
2. No "PST" , Factor Using Ratio or Trial and error method
3. Check Using FOIL

Example PST: $c^2 + 10c + 25 = (c + 5)^2$ For examples of # 2, See Ratio-Trial/Error Method GC*

Step 4

Four Terms: Grouping

1. Group two terms together that have a GCF.
2. Factor out the GCF from each pair. Look for common binomial.
3. Re-write with common binomial times other factors in a binomial.

Example: $5t^4 + 20t^3 + 6t + 24 = (t + 4)(5t^3 + 6)$

Step 5

Make Sure that each Polynomial is factored completely.
If you have tried steps 1 – 4 and the polynomial cannot be factored, the polynomial is Prime.

Why are these steps important?

Following these steps will allow one to factor any polynomial that is not prime.

Factoring allows one to find the x-intercepts and in turn graph the polynomial.

TOPIC

David Blackwell African American Mathematician

First African Amer. to chair (statistics) at a major university: University of California at Berkeley

First African Amer. named to The National Academy of Sciences

At age 22, earned a Ph. D from the University of Illinois

Wrote the classic "Theory of Games and Statistical Decisions"

First African Amer. to hold a permanent position at a major university: University of Calf. at Berkeley

Work considered within the "extraordinary" mathematician rank

Won the von Neumann Theory Prize (Operations Research Society of America)

Considered one of the greatest African American Mathematicians

Topic

Divisibility

Details

Dividing numbers that don't have remainders

Main Idea

Two (2)

Details

If a number ends with O, 2, 4, 6, 8

Main Idea

Five (5)

Details

If a number ends with O or 5

Main Idea

Ten (10)

Details

If a number ends with O

Main Idea

Three (3)

Details

If the sum of the digits can be divided by 3

Main Idea

Nine (9)

Details

If the sum of the digits can be divided by 9

Polynomial Products

Multiplying Polynomials and Special Products

Main idea: Polynomials times Polynomials

Monomial times a Polynomial:
Use the distributive property (x coeff + exponents)
Example:
$-4y^2 (5y^4 - 3y^2 + 2) = -20y^6 + 12y^4 - 8y^2$

Polynomial times a Polynomial:
Use the distributive Property
Example:
$(2x - 3) (4x^2 + x - 6) =$
$8x^3 - 10x^2 - 15x + 18$

Main idea: Binomials times Binomials

FOIL:
F = First
O = Outer
I = Inner
L = Last

Example:
$(3x - 5)(2x + 7)=$
$6x^2 + 11 x - 35$

Main idea: Square of a Binomial

$(a + b)^2 = a^2 + 2ab + b^2$

$(a - b)^2 = a^2 - 2ab + b^2$

1st term: Square the first term
2nd term: Multiply two terms and Double
3rd term: Square the last term

Examples:
$(x + 6)^2 =$
$x^2 + 12 x + 36$
$(7m - 2p)^2 =$
$49m^2 - 14mp + 4p^2$

Main idea: Difference of Two Squares

DOTS:
$(a + b) (a - b) = a^2 - b^2$

Example:
$(t^3 - 6) (t^3 + 6) =$
$t^6 - 36$

So what? What is important to understand about this?

AHSGE Objective: I -3. Applications include finding area and volume. Special products are used in graphing functions by hand. Punnett Squares in Biology.

Topic

Multiplying Powers

Main Idea

Same Base

Details

1. Multiply coefficients
2. Keep base
3. Add Exponents
4. Simplify

Main Idea

(Power) Power?

Details

1. Keep base
2. Multiply Exponents
3. Simplify

Main Idea

(Product) Power?

Details

1. Raise each factor to power
2. Simplify

Functions, Equations, and Graphs

Relations and Functions

- Domain
  Range
- Function – vertical line test
- Function notation/evaluating functions

Linear Models

- Scattergram
- Line of best fit or trend line
- Equation of line of best fit

Linear Equations

- Standard form
  $Ax + By = C$
- Slope-intercept form
  $y = mx + b$
- Point-slope form
  $y - y_1 = m(x - x_1)$

Absolute Value Functions

- Always graphs as a "V"
- $y = |x - h| + k$
  Vertex (h,k)
- h is horizontal movement and k is vertical movement

Direct Variation

- Constant of variation, k
- $y = kx$
- $\frac{y}{x} = \frac{y}{x}$

Graphing Two-Variable Inequalities

- Graph line first – called the boundary equation
- Test a point not on the line – best choice is (0,0)
- If test is true, shade to include point

Factor $A^2 - C^2$

These are the steps to ...

"DOTS" = Difference of Two Squares

Name

Five Steps

Step 1

Factor out the GCF.

Example: $3x^4 - 48$

$3(x^4 - 16)$

Step 2

Check for : 1.) 2 terms
2.) Minus Sign
3.) A and C are perfect squares

Step 3

Write two binomials

Signs: One + ; One - $3(\quad + \quad)(\quad - \quad)$

Step 4

Write the numbers and variables before they were squared in the binomials. (Note: Any even power on a variable is a perfect square. . . just half the exponent when factoring) $3(x^2 + 4)(x^2 - 4)$

Step 5

Check for DOTS in DOTS

$3(x^2 + 4)(x^2 - 4)$

$3(x^2 + 4)(x + 2)(x - 2)$

Check using FOIL.

Why are these steps important? .

Following these steps will allow one to factor any polynomial that is not prime.

Factoring a quadratic trinomial enables one to determine the x-intercepts of a parabola.

Factoring also enables one to use the x-intercepts to graph.

Key Topic

Lines

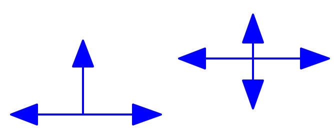
is about...

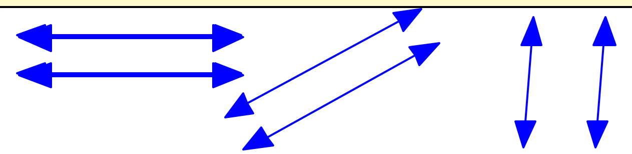
2 kinds of lines

Main idea	*Main idea*
Perpendicular	*Parallel*

Detail *Essential because...*

Intersect / meet	*never intersect, meet, or touch*
form right angles	*same plane*
forms a square corner	*go in same direction*





So what? What is important to understand about this?

There are only two types of lines

Key Topic

Angles

is about...

4 kinds of angles

Straight

- 180^{0}
- does not bend

Acute

- less than 90^{0}
- 1^{0} -- 89^{0}
- small angle

Right

- always 90^{0}
- square corners
- perpendicular

Obtuse

- greater than 90^{0}
- 91^{0} - 179^{0}
- wide angle
- angle larger than right angle

So what? What is important to understand about this?

type of angle is determined by the degree of arch

Functions, Equations, and Graphs

Direct Variation

- $\frac{y}{x} = \frac{y}{x}$
- y = kx
- Constant of variation, k

Graphing Two-Variable Inequalities

- If test is true, shade to include point
- Test a point not on the line – best choice is (0,0)
- Graph line first – called the boundary equation

Linear Equations

- Point-slope form $y - y_1 = m(x - x_1)$
- Slope-intercept form y = mx + b
- Standard form Ax + By = C

Absolute Value Functions

- h is horizontal movement and k is vertical movement
- y = |x – h| + k
  Vertex (h,k)
- Always graphs as a "V"

Relations and Functions

- Function notation /evaluating functions
- Function – vertical line test
- Domain
  Range

Linear Models

- Equation of line of best fit
- Line of best fit or trend line
- Scattergram