CURRICULUM GUIDE
to the Alabama Course of Study

Mathematics
PREREQUISITES FOR ALGEBRA I

- **General Education and Special Education Students: Preparation for Algebra I Academic Content Standards**
- **Special Education Students: Essential Skills for the Alabama Occupational Diploma**

Ed Richardson, State Superintendent of Education • Alabama Department of Education
Bulletin 2003 • No. 46

Curriculum Guide to the Alabama Course of Study: Mathematics Prerequisites for Algebra I & Essential Skills for the Alabama Occupational Diploma

Table of Contents

ACKNOWLEDGMENTS ii

Introduction 1

Course Selection 2

Functional Curriculum 2

Content Standard Correlation 2

ESSENTIAL MATHEMATICS I 3

ESSENTIAL MATHEMATICS II 6

ALGEBRAIC EXPLORATIONS I 12

ALGEBRAIC EXPLORATIONS II 21

APPENDIX

Course Sequence Options 31

Acknowledgments

This document was developed by the 2002-2003 Alabama Occupational Diploma Mathematics Special Education Task Force. The Task Force began work in May 2002 and the Alabama State Board of Education adopted this document at its (date of meeting) meeting.

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Alabama Occupational Diploma
Curriculum Guide to the Standards: Mathematics

Introduction

The Alabama Occupational Diploma (AOD), approved by the Alabama State Board of Education on May 8, 1997, enables students with disabilities to pursue a diploma option leading to skills for independent, competitive employment and/or possible further technical training. Since that time, the *Alabama Occupational Diploma Courses of Study* have been included in the *Alabama Courses of Study* general education documents for mathematics, science, English language arts, and social studies. However, the No Child Left Behind Act of 2001 requires all states to adopt challenging, academic mathematics content standards that apply to **all** students. For this reason, a separate mathematics curriculum that applies only to students pursuing the AOD will not be developed. A new document, the *Alabama Occupational Diploma Curriculum Guide to the Standards: Mathematics*, will provide challenging mathematics content through four levels of instruction that serve as a foundation for the Grades 9-12 mathematics courses identified in the *Alabama Course of Study: Mathematics* (Bulletin 2003, No. 4).

This new guide is divided into four levels. Students should progress through the levels at their own pace, beginning at the level that meets their individual instructional needs, and progressing into the appropriate mathematics course of the *Alabama Course of Study: Mathematics* (Bulletin 2003, No. 4).

Level I of this guide is entitled Essential Mathematics I. This course provides students with the opportunity to learn basic operations involving whole numbers, decimals, and fractions. Consumer-related skills at this level include computing change and developing check-writing skills. Basic geometry skills at this level focus on measurement and identification of geometric shapes and representations.

Level II, entitled Essential Mathematics II, focuses on basic operations involving decimals and percents, laws of exponents, and adding and subtracting to determine elapsed time. Consumer-related skills include reconciling bank statements and computing pay rates. Additional content involves identifying basic functions and performing basic graphing skills. Basic geometry skills include determining perimeter and area of various shapes and measuring angles.

Level III is Algebraic Explorations I. This level focuses on basic operations involving integers and numerical expressions. Consumer-related skills include calculating taxes, while geometry skills include complementary and supplementary angles, and the measurement of circles and surface area of rectangular solids. Data reporting, interpreting data, and solving proportions are also addressed at this level.

The Level IV curriculum, entitled Algebraic Explorations II, provides students with an exploration of basic algebraic concepts such as graphing and analyzing linear equations, solving problems using the Pythagorean Theorem, and deducing relationships including congruence and similarity. Consumer-related skills focus on problems that involve budgets, loans, credit purchases, and measurement. Upon successful completion of Level IV, students are prepared to enroll in the Algebra I course outlined in the *Alabama Course of Study: Mathematics* (Bulletin 2003, No. 4).

Course Selection

The *Alabama Occupational Diploma Curriculum Guide to the Standards: Mathematics* is designed to help students move from the point at which they enter the Alabama Occupational Diploma program in Grade 9 through a sequence of four levels of courses that prepare them to be able to pursue the Grades 9-12 general education mathematics curriculum. Some students may meet the requirements to participate in the general education mathematics curriculum, while others may follow the four levels of courses in the *Alabama Occupational Diploma Curriculum Guide to the Standards: Mathematics*.

Based upon the current level of performance in mathematics, students may enter one of the four levels of courses at any point the Individual Educational Program (IEP) team deems appropriate. For example, students who only need some additional Pre-Algebra preparation before entering the Algebra I course would begin with Algebraic Explorations II. Other students who enter Grade 9 with a basic knowledge of whole number computation and basic fraction skills would begin with Essential Mathematics I in order to increase their computational fluency in whole number computation, fraction skills, and basic geometry skills.

Functional Curriculum

The *Alabama Occupational Diploma Curriculum Guide to the Standards: Mathematics* is designed to meet the functional and workplace skills that youth with disabilities need to in order to meet employment and other postschool options. Money-related skills, checking and savings accounts, budgets, wages, taxes, and consumer-related problems are incorporated into all four course levels. Coordination with the Career/Technical Education curriculum to demonstrate the relationship between mathematical concepts and their real-world applications is encouraged and should be developed.

Content Standard Correlation

The *Alabama Occupational Diploma Curriculum Guide to the Standards: Mathematics* is correlated to the Algebra I and Geometry content standards located in the *Alabama Course of Study: Mathematics* (Bulletin 2003, No. 4). Each content standard from these two courses is listed before the benchmarks that comprise the content of the four levels of courses found in this curriculum guide. The AOD benchmarks identify the prerequisite skills that students must master in order to be successful in the Algebra I and Geometry courses described in the *Alabama Course of Study: Mathematics* (Bulletin 2003, No. 4).

Essential Mathematics I

The major focus of Essential Mathematics I is on the development of student fluency in basic mathematical skills. Concepts addressed at this level of instruction reinforce skills taught in previous grades. This course is designed for students who need additional help in acquiring these prerequisite skills necessary to progress to either the general education mathematics curriculum or to advance to the next course level identified in the *Alabama Occupational Diploma Curriculum Guide to the Standards: Mathematics*. Essential Mathematics I focuses on basic operations of whole numbers, decimals, and fractions; measurement; and identification of geometric terms. An emphasis on functional skills is included by providing instruction in computing change and writing checks.

Due to the unique needs of students with disabilities, the instructional content of Essential Mathematics I addresses the different learning modalities of each individual student. The learning environment allows students to utilize hands-on materials, manipulatives, and other resources as well as different types of technology currently available. Instruction at this level focuses on integrating problems into real-life applications as much as possible in order to make learning more relevant for students.

Students enrolled in Essential Mathematics I must possess certain prerequisite skills. These skills include basic computational skills with whole numbers, calculator skills, time-telling skills, the ability to recognize and name common shapes and figures, and the ability to read and write whole numbers. The IEP team should examine the student's current level of performance in these areas before determining placement in this course.

Number and Operations

Algebra Content Standard 1: **Simplify numerical expressions using properties of real numbers and order of operations, including those involving square roots, radical form, or decimal approximations.**

Benchmark 1: Determine place values through millions and decimals through ten thousandths.

Benchmark 2: Demonstrate proficiency in basic operations on whole numbers and decimals.

- Using basic operations to solve word problems involving money values

Benchmark 3: Use basic operations to solve problems involving fractions with like and unlike denominators, including mixed numbers.

- Determining fractional parts of whole objects including 1/2, 1/3, 1/4, 1/8, and 1/16
- Determining equivalent fractions
- Ordering fractions
- Using greatest common factor (GCF) to simplify fractions

Benchmark 4: Solve numerical and monetary problems using estimation.

- Decimal numbers to the nearest thousandths
- Money values to the nearest dollar and/or nearest dime
- Whole numbers to millions

Benchmark 5: Compute change in consumer-related problems.

Benchmark 6: Demonstrate the ability to write checks and maintain a checkbook.

- Completing deposit slips

Algebra

<u>Algebra Content Standard 2</u>: Analyze linear functions from their equations, slopes, and intercepts.

Benchmark 7: Recognize numbers less than zero on a number line.

<u>Algebra Content Standard 7</u>: Solve multistep equations and inequalities including linear, radical, absolute value, and literal equations.

Benchmark 8: Demonstrate an understanding of the concepts of *greater than*, *less than*, and *equal to* when comparing simple number sentences.

Example:

$$\begin{array}{rcl} (2 \times 5) + 6 & & (9 \div 3) + 5 \\ 10 + 6 & & 3 + 5 \\ 16 & > & 8 \end{array}$$

Geometry

<u>Geometry Content Standard 2:</u> Justify theorems related to pairs of angles, including angles formed by parallel and perpendicular lines, vertical angles, adjacent angles, complementary angles, and supplementary angles.

Benchmark 9: Identify an angle as straight, acute, obtuse, or right.

<u>Geometry Content Standard 3</u>: Verify the relationships among different classes of polygons by using their properties.

Benchmark 10: Find the perimeter and area of quadrilaterals.

- Using simple word problems
  Example: Bart is fencing in his yard. The dimensions of his yard are 100 feet by 200 feet. How many feet of fencing will Bart need?

  Example: Li is retiling her bathroom floor. The dimensions of the floor are 8 feet by 10 feet. How many one-foot square tiles will Li need in order to retile the bathroom?

Benchmark 11: Find the volume of cubes.

Example: Jorge works for a packing company. He has a shipping box shaped in a cube that has 14 inch sides. He will be packaging gift boxes (shaped in cubes that have 2 inch sides) into the shipping box. How many gift boxes will he be able to fit inside the shipping box?

Step One: Find the volume of the shipping box. (Volume of shipping box: 14″ x 14″ x 14″ = 2744 cubic inches.)

Step Two: Find the volume of the gift box. (Volume of gift box: 2″ x 2″ x 2″ = 8 cubic inches.)

Step Three: Divide the volume of the shipping box (2744 cubic inches) by the volume of the gift box (8 cubic inches) to determine the number of gift boxes that will fit into shipping box. (Answer: 2744 cu. in. ÷ 8 cu. in. = 343.)

Solution: Jorge can place 343 gift boxes into the shipping box.

Benchmark 12: Identify the relationship of geometric representations to real-life objects.

- Lines
- Line segments
- Rays
- Parallel lines
- Perpendicular lines

Example: Given the floor plan of a house, identify the geometric representations listed above on the floor plan.

Benchmark 13: Identify plane and solid figures and the terms associated with them.

- Side
- Surface
- Edge
- Line
- Cube
- Cone
- Cylinder
- Sphere
- Polygon

Benchmark 14: Use a ruler to determine customary length and width to the nearest 1/16 of an inch, and metric length and width to the nearest millimeter.

Essential Mathematics II

Essential Mathematics II focuses on the expansion of student mathematical fluency in the areas of number and operations and in the area of geometry. This course builds on content included in Essential Mathematics I and also provides students with an introduction to Pre-Algebra. Essential Mathematics II is designed for students who already possess fluency in basic mathematical concepts, but need to expand their application of these mathematical concepts to a more advanced level. The content focuses on basic operations involving decimals and percents and laws of exponents as well as addition and subtraction to determine elapsed time. An emphasis on expanding functional skills in the areas of bank statement reconciliation and pay rate computation is also incorporated into the content. Pre-Algebra skills include identification of basic functions and an introduction to basic graphing. Geometry skills include determining perimeter and area of various shapes and determining measures of angles.

Due to the individual learning styles unique to special needs learners, the instructional content of Essential Mathematics II addresses all types of learning styles: kinesthetic, visual, auditory, and tactile. To the greatest extent possible, classroom instruction incorporates opportunities for student use of manipulatives and technology to demonstrate learning. Instruction at this level continues to provide an emphasis on integrating mathematical problems into real-life applications, thereby allowing students to relate course content to their everyday lives.

Students enrolled in Essential Mathematics II must possess the prerequisite skills addressed in Essential Mathematics I. Special emphasis is placed on students having acquired the prerequisite skills of being familiar with decimal place value concepts, plotting points on a number line, finding area and perimeter of regular quadrilaterals, filling out checks, performing basic operations on fractions, measuring with a ruler to the nearest 1/16 of an inch or to the nearest millimeter, and identifying different types of angles. The student's current level of performance should be closely examined by the IEP team to determine appropriate course placement.

Number and Operations

<u>Algebra Content Standard 1</u>: **Simplify numerical expressions using properties of real numbers and order of operations, including those involving square roots, radical form, or decimal approximations.**

Benchmark 1: Use basic operations to solve problems involving decimals and percents.

- Comparing rational numbers written as decimals or percents
- Converting among fractions, decimals, mixed numbers, and percents
- Using different forms of notation to symbolize ratios and rates
  Examples: 3 to 4, 3/4, 3:4

- Applying the distributive property to compute with rational numbers
  Example:

$$\begin{aligned} 2(3.5 + 6) - 1/2 &= \\ 2(3.5) + 2(6) - 1/2 &= \\ 7 + 12 - 1/2 &= \\ 19 - 1/2 &= 18\ 1/2 \end{aligned}$$

- Determining discounts, sale price, sales tax, shipping charges, and fees

Benchmark 2: Simplify expressions containing natural number exponents by applying one or more of the laws of exponents.

- Writing numbers using scientific notation
  Example: $120{,}000 = 1.2 \times 10^5$

- Evaluating powers
  Example: $7^3 = 7 \times 7 \times 7 = 343$

Benchmark 3: Reconcile bank statements.

- Check charges and fees

Benchmark 4: Use the basic operations of addition and subtraction to determine elapsed time.

Benchmark 5: Calculate total salary for a given pay period.

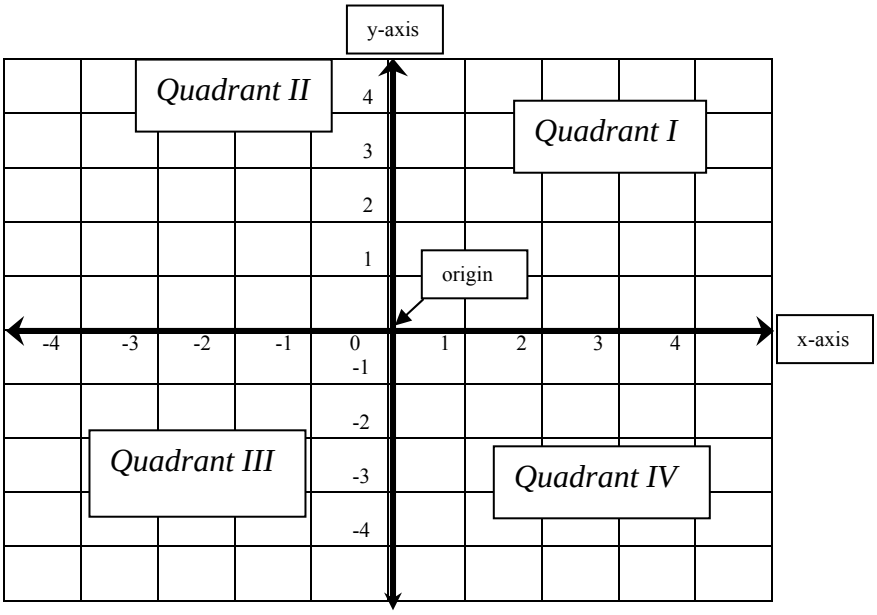
- Calculating net pay
- Calculating time-and-a-half and double-time rates
- Calculating commissions

Algebra

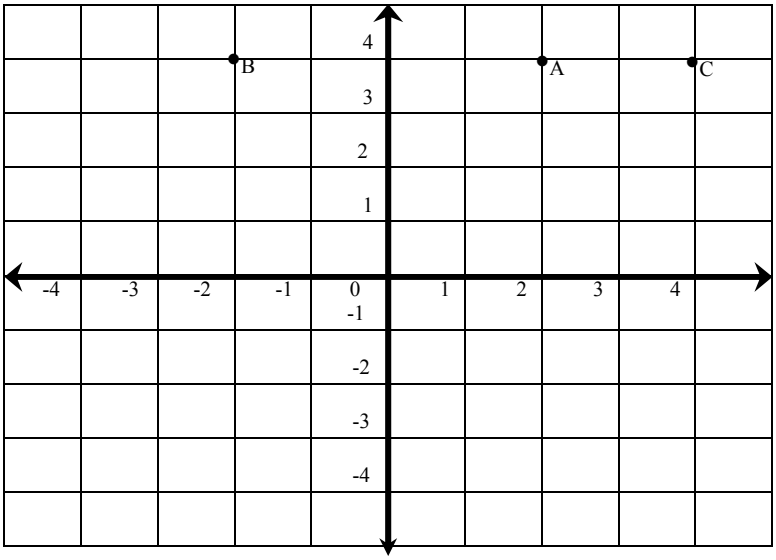
<u>Algebra Content Standard 2</u>: Analyze linear functions from their equations, slopes, and intercepts.

Benchmark 6: Plot coordinates on grids, graphs, and maps.

- Identifying components of the Cartesian plane including the x-axis, y-axis, origin, and quadrants
  Example:



- Identifying the coordinates of a point on the Cartesian plane
  Examples: (a) A = (2,4); B = (-2,4); C = (4,4)



(b) The detectives from the Morgan City Police Department mapped a grid plot at the scene of a crime in order to document location of the criminal evidence. The coordinate location of the evidence is (5,4), (6,7), (1,3), and (-1,4). Draw a Cartesian coordinate system. Plot the points for the location of the criminal evidence.

<u>Algebra Content Standard 3</u>: Determine characteristics of a relation, including its domain, range, and whether it is a function, when given graphs, tables of values, mappings, or sets of ordered pairs.

Benchmark 7: Identify functions from information in tables, sets of ordered pairs, and mappings.

Examples: tables–

x	y
2	8
3	9
4	10
5	11

(function)

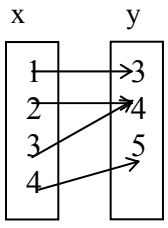
x	y
2	6
3	7
2	9
3	11

(not a function)

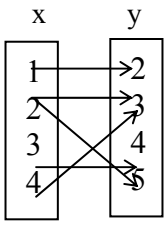
sets of ordered pairs– (1,4), (3,6), (2,5) (function)

(2,5), (3,6), (2,7) (not a function)

mappings–



(function)



(not a function)

Geometry

<u>Algebra Content Standard 10</u>: Calculate length, midpoint, and slope of a line segment when given coordinates of its endpoints on the Cartesian plane.

Benchmark 8: Determine the distance between two points on a scale drawing or a map, using a ruler.

<u>Geometry Content Standard 2</u>: Justify theorems related to pairs of angles, including angles formed by parallel and perpendicular lines, vertical angles, adjacent angles, complementary angles, and supplementary angles.

Benchmark 9: Measure angles using a protractor.

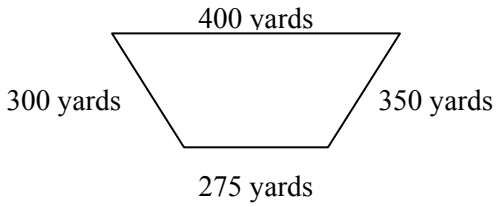
- Estimating angle measurements

<u>Geometry Content Standard 3</u>: Verify the relationships among different classes of polygons by using their properties.

Benchmark 10: Find perimeter and area of triangles, trapezoids, and parallelograms.

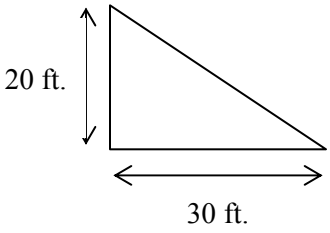
- Using simple word problems

  Examples: perimeter– The park manager is building a stage in the shape of a trapezoid. Use the following diagram to determine the length of fence that will be needed to enclose the stage.



P = a + b + c + d
P = 400 + 350 + 350 + 275
P = 1375 yards

area– A gardener is planting a flowerbed in the corner of the yard. Use the diagram below to determine the area of the garden.



A = 1/2 x b x h
A = 1/2(30)(20)
A = 300sq. ft.

<u>Geometry Content Standard 5</u>: Solve real-life and mathematical problems using properties and theorems related to circles, quadrilaterals, and other geometric shapes.

Benchmark 11: Identify parts of a circle.

- Center
- Radius
- Diameter
- Chord

<u>Geometry Content Standard 8</u>: Deduce relationships between two triangles, including proving congruence or similarity of the triangles from given information, using the relationships to solve problems and to establish other relationships.

Benchmark 12: Identify proportions as true or untrue.

Example:

$\frac{3}{5} = \frac{15}{25}$ (true) $\frac{4}{10} = \frac{10}{40}$ (untrue)

Data Analysis and Probability

<u>Algebra Content Standard 12</u>: Compare various methods of data reporting, including scatterplots, stem-and-leaf plots, histograms, box-and-whisker plots, and line graphs, to make inferences or predictions.

Benchmark 13: Identify mean, median, mode, and range of a given set of numbers.

<u>Algebra Content Standard 15</u>: Estimate probabilities given data in lists or graphs.

Benchmark 14: Find the probability of a simple event.

- Expressing probability as ratios, percents, or decimals
  Example: John has two additional concert tickets. John has five brothers (Mark, David, Carl, Eric, and Matt). What is the probability that Mark and David will be chosen to receive the additional tickets?

Algebraic Explorations I

The major focus of Algebraic Explorations I is to expand the Pre-Algebra skills acquired by students in Essential Mathematics II and to increase student application of geometric concepts. This level is designed for students who have mastered the essential mathematical skills needed for functioning in society and who are ready to advance their skill levels into the areas of Algebra and higher-level geometric theory. Algebraic Explorations I places emphasis on Pre-Algebra skills such as basic operations involving integers and numerical expressions, algebraic expressions, functions, and linear equations. Geometric concepts include angle measurements, measurement of circles, and measurement of surface area of rectangular solids. Measurement skills are incorporated into this course to help students understand the relationship between measurements and their function in the world of work. Data analysis is addressed through student calculation of measures of central tendency as well as interpretation of this information from various graphical displays.

The learning environment provides students with opportunities to utilize real-world experiences and relevant Career/Technical Education program applications that demonstrate the use of mathematics in society and the workplace. A greater emphasis on the use of technology such as scientific calculators is a focus of instruction. Hands-on manipulatives and other resources are also used to facilitate the learning modalities of individual students.

Students enrolled in Algebraic Explorations I must demonstrate certain prerequisite skills. These skills include the use of integers in plotting points on number lines and Cartesian planes; the ability to identify proportions; the use of decimals and percents; an understanding of the order of operations; a familiarity with identifying functions from sets of ordered pairs, tables, and mappings; and a knowledge of scientific calculator skills. Further knowledge of angle measurement, the meaning of variables, a basic knowledge of how to solve formulas, and knowledge of terms associated with circles must be understood by the students. The ability to understand measures of central tendencies, to read and interpret various graphic displays, and the ability to compute simple probability are also required prerequisite skills. It is essential that the IEP team examine the student's current level of performance when determining placement in this course.

Number and Operations

Algebra Content Standard 1: **Simplify numerical expressions using properties of real numbers and order of operations, including those involving square roots, radical form, or decimal approximations.**

Benchmark 1: Use basic operations to solve numerical problems involving integers.

Example: Michael had a balance of $35 in his checking account. A debit card transaction of $52 that he forgot to record in his check register cleared the bank last night. The bank has now charged him a $29 insufficient funds fee. What is the current balance in Michael's checking account?

Benchmark 2: Solve proportions.

- Converting equivalent measures within a system using a conversion chart
- Comparing prices of items of similar quality to the determine most economical buy
  Example: Two stores are selling identical sweaters. One store is advertising the sweaters at 2 for $29.99. The other store is selling the sweaters for $14.50 each. Which store has the most economical buy on the sweaters?

Benchmark 3: Use order of operation to evaluate numerical expressions.

- Computing absolute values
  Examples:

$\|-4\| + 6$	=		$2\|-6\|$	=	
$4 + 6$	=	10	$2(6)$	=	12

- Computing perfect squares and square roots through 225
  Examples: perfect squares– $3^2 = 9$, $5^2 = 25$
  square roots– $\sqrt{36} = 6$, $2\sqrt{36} = 2 \times 6 = 12$
- Applying properties of operations to compute with integers, fractions, and decimals
- Using inverse properties of addition and multiplication
- Computing irrational numbers
  Examples: $\sqrt{3} = 1.7320508$, $\sqrt{11} = 3.31662447$

Benchmark 4: Calculate taxes and/or refunds.

- Using 1040EZ or 1040A federal forms
- Using 40A state forms

Algebra

<u>Algebra Content Standard 3</u>: Determine characteristics of a relation, including its domain, range, and whether it is a function, when given graphs, tables of values, mappings, or sets of ordered pairs.

Benchmark 5: Classify variables in a function as independent or dependent.

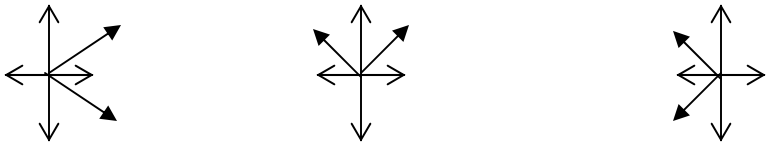
- Identifying functions from equations and graphs
  Examples: equations– Which of the equations is represented by the data in the table?

Table	
x	y
0	1
1	4
-1	-2

Equations
$y = 3x - 1$
$y = 3x + 1$
$y = 2x - 2$
$y = 2x + 1$

(Answer: $y = 3x + 1$)

graphs– Which of these graphs represents a function? (Answer: Graph B)



A B C

- Identifying the rule that defines a function
- Finding the range of functions when given the domain
  Example: Find the range of $f(x) = 2x^2 + x$, where the domain is {-3, -2, -1, 0, 1, 2, 3}.

$f(-3) = 2(-3)^2 + (-3) = 15$ $f(0) = 2(0)^2 + (0) = 0$
$f(-2) = 2(-2)^2 + (-2) = 6$ $f(1) = 2(1)^2 + (1) = 3$
$f(-1) = 2(-1)^2 + (-1) = 1$ $f(2) = 2(2)^2 + (2) = 10$
$f(3) = 2(3)^2 + (3) = 21$

(Range is {0, 1, 3, 6, 10, 15, 21}.)

Benchmark 6: Classify relations as linear or nonlinear by examining their graphs.
Examples:

linear nonlinear

<u>Algebra Content Standard 5</u>: Perform operations of addition, subtraction, and multiplication on polynomial expressions.

Benchmark 7: Use order of operations to evaluate and simplify algebraic expressions.

Examples: (a)

$3x^2 + 5x - x^2 - 2x + 6 =$

$3x^2 - x^2 + 5x - 2x + 6 =$

$2x^2 + 3x + 6 =$

(b) Evaluate $k + 30$ where $k = 10$.
(Answer: $k + 30 = 10 + 30 = 40$)

(c) Beondra earns $110 per week working 20 hours per week at a local grocery store. She wants to earn an additional $135 per week so that she can buy a car. If Beondra gets a second job working 10 hours per week, how much will Beondra need to earn per hour at her second job?

Benchmark 8: Solve simple one- and two-step linear equations.

Example: Jamal is working on a project for his science class. He knows the current temperature is 90 degrees Fahrenheit. However, he needs to convert the temperature to degrees Celsius for his science project. Use the equation $F = \frac{9}{5}(C + 2)$ to determine the current temperature in degrees Celsius that Jamal needs to use for his science project.

<u>Algebra Content Standard 6</u>: Factor binomials, trinomials, and other polynomials using GCF, difference of squares, perfect square trinomials, and grouping.

Benchmark 9: Identify an algebraic expression as being a binomial or trinomial.

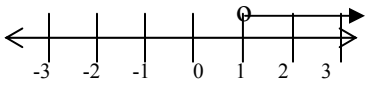
Examples: binonmial– $4x^2 + x$

trinomial– $4x^2 + x + 3$

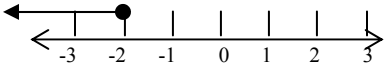
<u>Algebra Content Standard 7</u>: Solve multistep equations and inequalities including linear, radical, absolute value, and literal equations.

Benchmark 10: Determine solution sets of inequalities on a number line.

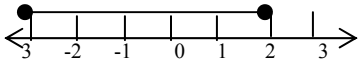
Examples: (a) Given $x > 1$, represent the inequality on a number line.



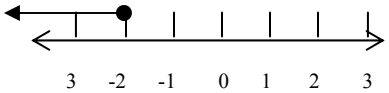
(b) Given $x \leq -2$, represent the inequality on a number line.



(c) Given $-3 \leq x \leq 2$, represent the equality on a number line.



(d) What is the solution set for the graph below?



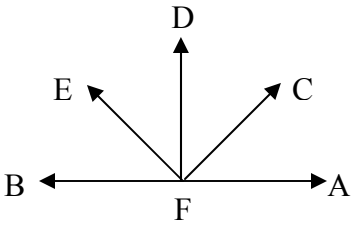
(Answer: $x \leq -2$)

Geometry

<u>Geometry Content Standard 2</u>: Justify theorems related to pairs of angles, including angles formed by parallel and perpendicular lines, vertical angles, adjacent angles, complementary angles, and supplementary angles.

Benchmark 11: Determine the measures of special angle pairs, including adjacent, vertical, supplementary, complementary, and angles formed by parallel lines cut by a transversal.

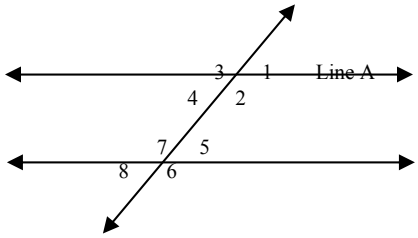
Examples: (a)



m∠CFA = 45°
m∠DFA = 90°
m∠EFA = 120°

Name two complementary angles. (Answer: ∠DFC and ∠CF)
Name two supplementary angles. (Answer: ∠EFA and ∠EFB)

(b)



If m∠2 is 150°, what is the measure of ∠3? (Answer: m ∠3 is 150° because ∠2 and ∠3are vertical angles.)

If m∠2 is 150°, what is the measure of ∠1? (Answer: ∠2 and ∠1 are adjacent angles, so m∠2 + m∠1 = 180° because ∠2 and ∠1 are supplementary.)

$$\begin{aligned} 150° + m\angle 1 &= 180° \\ \underline{-150°} &\quad \underline{-150°} \\ m\angle 1 &= 30° \end{aligned}$$

If m∠2 = 150°, what is the m∠6?

$$\begin{aligned} m\angle 2 &= m\angle 6 \\ m\angle 6 &= 150° \end{aligned}$$

Theorem: If two lines are cut by a transversal, then the corresponding angles are congruent.

Geometry Content Standard 5: Solve real-life and mathematical problems using properties and theorems related to circles, quadrilaterals, and other geometric shapes.

Benchmark 12: Solve problems involving circumference, diameter, and area of circles.

- Using appropriate units of measure to describe circumference, diameter, and area
- Determining circumference, diameter, and area by measuring using customary and metric units
- Using circumference and diameter to approximate the value of *pi* (π)
- Identifying *pi* (π) as an irrational number
- Using formulas for circumference, diameter, and area
  Example: Phillipe and his wife recently purchased a circular hot tub. They have decided to purchase a cover to keep leaves and other debris out of the hot tub. The diameter of the hot tub is 5 feet. Find the surface area of the hot tub in order to determine the size of the cover they need to buy.

Geometry Content Standard 8: Deduce relationships between two triangles, including proving congruence or similarity of the triangles from given information, using the relationships to solve problems and to establish other relationships.

Benchmark 13: Select the appropriate measurement of length, height, width, weight, and capacity for real-life objects.

Examples: (a) If you sow a field with grass seed, which measurement would you use to determine the area of the field? (Answer: C)

A. gallons
B. liters
C. square yards
D. inches

(b) If you are paving a road, which measurement would you use to determine the length of the road? (Answer: B)

A. meters
B. kilometers
C. inches
D. pints

(c) When baking bread, which measurement would you use to determine the amount of yeast needed? (Answer: B)

A. pints
B. ounces
C. liters
D. kiloliters

Measurement

Geometry Content Standard 16: Calculate surface areas and volumes of solid figures, including spheres, cones, and pyramids.

Benchmark 14: Compute the surface area of rectangular solids.

Example: Enrique is building an addition on his boathouse. The addition is 12 feet wide, 15 feet long, and 7 feet high. What is the total surface area of the addition Enrique is building?

Data Analysis and Probability

Algebra Content Standard 12: Compare various methods of data reporting, including scatter plots, stem-and-leaf plots, histograms, box-and-whisker plots, and line graphs, to make inferences or predictions.

Benchmark 15: Calculate the mean, median, mode, and range using a given set of data or graphs including histograms, frequency tables, and stem-and-leaf plots.

Example: Ken is employed as a produce buyer for a local restaurant. Ken has recorded the number of pounds of potatoes the restaurant uses each day over a month's time and has entered this data into the table shown below. Use the information recorded in the table to determine the mean amount of potatoes used each day. Then determine the mode of the number of pounds of potatoes used.

Pounds of Potatoes Used in January

Week	Number of Pounds on Monday	Number of Pounds on Tuesday	Number of Pounds on Wednesday	Number of Pounds on Thursday	Number of Pounds on Friday
Week 1	6	5	7	15	30
Week 2	4	5	8	22	35
Week 3	5	4	6	25	32
Week 4	2	6	9	21	33

Algebra Content Standard 13: Identify characteristics of a data set, including measurement or categorical and univariate or bivariate.

Benchmark 16: Interpret data from populations, using given and collected data.

- Representing the data with the most appropriate graph
- Making predictions by estimating the line of best fit from a scatter plot
- Comparing data sets involving two populations

  Example: A physical education teacher at an elementary school wants to find out if the number of sit-ups a student can do is correlated to the student's age. She compared her third-grade class with her fifth-grade class. In the data shown below, the first number represents each student's age, and the second number represents the number of sit-ups the student is able to perform. Use the data to plot the points on a scatter plot. Draw a line of best fit to determine if the data shows any correlation between a student's age and the number of sit-ups performed.

  Third-grade class: (8, 14), (9, 40), (8, 22), (8, 30), (9, 25)
  Fifth-grade class: (10, 12), (11, 50), (10, 35), (11, 19), (10, 20)

<u>Algebra Content Standard 15</u>: Estimate probabilities given data in lists or graphs.

Benchmark 17: Determine the probability of a compound event.

- Representing outcomes as a list, chart, picture, or tree diagram
- Determining the number of possible outcomes by using the fundamental counting principle or other technique

  Example: A temporary employment agency has four different jobs with four qualified workers for all the jobs. How many different ways could the workers be assigned to the jobs? (Answer: 4 x 3 x 2 x 1 = 24)

Algebraic Explorations II

Algebraic Explorations II is a continuation of the Pre-Algebra curriculum presented in Algebraic Explorations I. This course is designed to meet the needs of those students who require additional opportunities to develop the prerequisite skills necessary to be successful in the Algebra I course of the *Alabama Course of Study: Mathematics* (Bulletin 2003, No. 4). Algebraic Explorations II places emphasis on basic algebraic concepts such as graphing and analyzing linear equations, evaluating polynomials, using the Pythagorean Theorem to solve problems, and deducing relationships that include congruence and similarity. Consumer-related skills include problems involving budgets, loans, credit purchases, and measurement. Upon successful completion of Algebraic Explorations II, students are prepared to enroll in the Algebra I course.

Instruction incorporates the application of mathematics in real-world situations as well as situations regarding content applicable to the Career/Technical Education program. Student use of technology such as scientific calculators and computers is an essential component of learning. Manipulatives and relevant community resources are incorporated into the learning environment as much as possible.

Students enrolled in Algebraic Explorations II must be able to demonstrate certain prerequisite skills. These skills include fluency in basic operations on integers; knowledge of linear functions; the ability to evaluate one- and two-step algebraic expressions; the ability to identify different types of polynomials; and the ability to calculate exponents, square roots, and absolute values. It is imperative that the IEP team closely examine the student's current level of performance before placement in this course.

Number and Operations

Algebra Content Standard 1: **Simplify numerical expressions using properties of real numbers and order of operations, including those involving square roots, radical form, or decimal approximations.**

Benchmark 1: Solve word problems involving the use of integers.

Example: At 6:00 a.m., the temperature was 35 degrees. It had risen 15 degrees by noon, but had fallen 10 degrees by 5:00 p.m. What was the temperature at 5:00 p.m.?

$$35° + 15° - 10° =$$
$$50° - 10° = 40°$$

Algebra

<u>Algebra Content Standard 2</u>: Analyze linear functions from their equations, slopes, and intercepts.

Benchmark 2: Graph linear relations by plotting points or by using the slope and y-intercept.

- Calculating the slope of a linear relation given tables or graphs

  Example: Formula for slope is $m = \frac{y_2 - y_1}{x_2 - x_1}$

x	y
0	1
1	0
2	-1

$(x_1, y_1) = (0,1)$
$(x_2, y_2) = (1,0)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-1}{1-0} = \frac{-1}{1}$$

- Determining x-intercepts and y-intercepts of lines

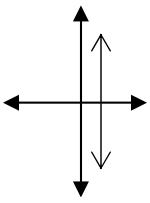
  Example: Find the y-intercept of the graph y = 2x + 1.
  Let x = 0.

  y = 2x + 1
  y = 2(0) + 1
  y = 1

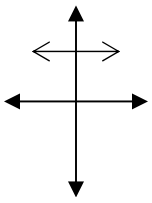
<u>Algebra Content Standard 4</u>: Represent graphically common relations, including x = constant, y = constant, y = x, y= $\sqrt{x}$, y = x^2, and y = |x|.

Benchmark 3: Represent graphically common relations, including x = constant, y = constant, y = x, y = $\sqrt{x}$, y = x^2, and y = |x|.

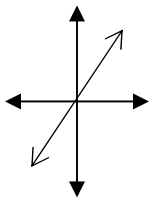
Examples:



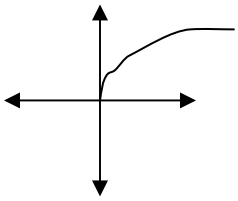
x = constant



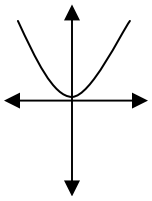
y = constant



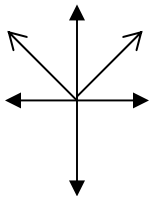
y = x



y = $\sqrt{x}$



y = x^2



y = |x|

<u>Algebra Content Standard 5</u>: Perform operations of addition, subtraction, and multiplication on polynomial expressions.

Benchmark 4: Solve problems involving operations of addition, subtraction, and multiplication on polynomials.

Examples:

addition– $2y^2 + 5y + 8$
$+ \underline{6y + 3}$
$2y^2 + 11y + 11$

subtraction– $(5y^2 + 8y + 10) - (y^2 + 9y + 7)$
$5y^2 + 8y + 10$
$- \underline{y^2 - 9y - 7}$
$4y^2 - y + 3$

multiplication– $(6b)(2b^2 + 2)$
$2b^2 + 2$
$\times \underline{6b}$
$12b^3 + 12b$

$(3x^2 + 2)(2x + 5)$
$6x^3 + 15x^2 + 4x = 10$

<u>Algebra Content Standard 6</u>: Factor binomials, trinomials, and other polynomials using GCF, differences of squares, perfect square trinomials, and grouping.

Benchmark 5: Factor binomials, trinomials, and other polynomials using GCF, differences of squares, perfect square trinomials, and grouping.

Examples:

(a) $3x^3 - 12x^2 + 0x$
$3x(x^2 - 4x + 3)$
$3x(x - 3)(x - 1)$

(b) $x^2 + 2x - 15$
$(x + 5)(x - 3)$

(c) $x^2 - 25$
$(x + 5)(x - 5)$

<u>Algebra Content Standard 7</u>: Solve multistep equations and inequalities including linear, radical, absolute value, and literal equations.

Benchmark 6: Translate verbal phrases into algebraic expressions and algebraic expressions into verbal phrases.

Examples: verbal to algebraic– four times the sum of a number and sixteen $\rightarrow$ $4(x + 16)$
algebraic to verbal– $3x + 6 \rightarrow$ three times a number plus six

Benchmark 7: Solve consumer-related problems involving loans and credit purchases.

- Computing the amount owed on simple interest loans
- Computing interest on credit purchases
- Using mortgage rate tables to determine monthly payments
- Using simple interest

Benchmark 8: Solve consumer-related problems related to measurement.

- Using distance, rate, and time formula
- Computing mileage

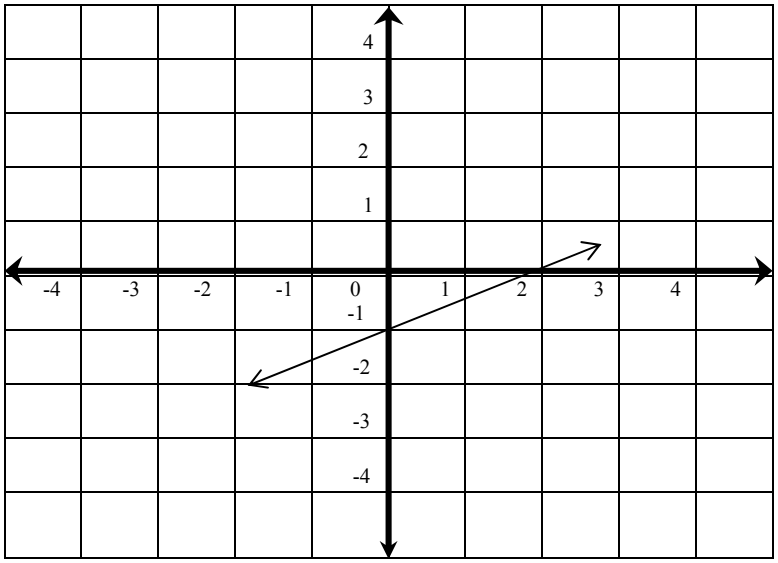
<u>Algebra Content Standard 8</u>: Solve systems of linear equations and inequalities in two variables graphically or algebraically.

Benchmark 9: Solve one- and two-step equations in two variables for the dependent variable in order to graph their solutions.

Example:

$$\begin{aligned} -x + 2y &= -2 \\ \underline{+x} &\quad \underline{+x} \\ \frac{2y}{2} &= \frac{x-2}{2} \\ y &= 1/2x - 1 \end{aligned}$$

y-intercept is -1 x-intercept is 2 slope is 1/2



<u>Algebra Content Standard 9</u>: Solve quadratic equations using the zero product property.

Benchmark 10: Use the zero product property to approximate numerical solutions to quadratic equations.

Example: Solve $x^2 + 3x - 10 = 0$.

$$(x = 5)(x - 2) = 0$$

$$\begin{aligned} x + 5 &= 0 & x - 2 &= o \\ -5 &= -5 & +2 &= +2 \\ x &= -5 & x &= 2 \end{aligned}$$

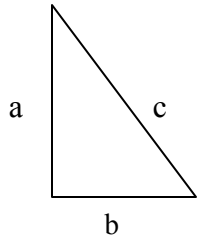
(Answer: -5, 2)

Geometry

<u>Geometry Content Standard 6</u>: Apply the Pythagorean Theorem to solve application problems, expressing answers in either simplified radical form or as decimal approximations, using Pythagorean triples when applicable.

Benchmark 11: Solve problems using the Pythagorean Theorem.

- Verifying the Pythagorean Theorem
- Applying the Pythagorean Theorem to determine if a triangle is a right triangle
- Applying the Pythagorean Theorem to find the missing length of a side of a right triangle
  Example: A 10-foot ladder is leaning against a house. The base of the ladder is 6 feet away from the house. How many feet up the house does the ladder reach?



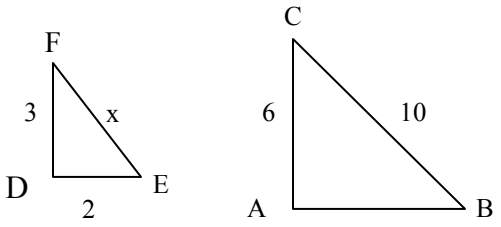
c = 10 ft.
b = 6 ft.

$$
\begin{aligned}
a^2 + b^2 &= c^2 \\
a^2 + (6)^2 &= (10)^2 \\
a^2 + 36 &= 100 \\
\underline{-36} &= \underline{-36} \\
a &= 64 \\
\sqrt{a^2} &= \sqrt{64} \\
a &= 8 \text{ ft.}
\end{aligned}
$$

<u>Geometry Content Standard 8</u>: Deduce relationships between two triangles, including proving congruence or similarity of the triangles from given information, using the relationships to solve problems and to establish other relationships.

Benchmark 12: Determine the lengths of missing sides and measures of angles in similar and congruent figures.

- Applying proportional reasoning
  Examples: (a) congruent segments–



$\triangle DEF$ is similar to $\triangle ABC$

Find x.

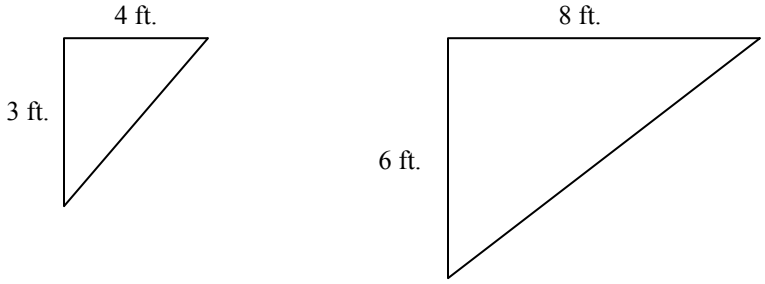
$$\frac{3}{6} = \frac{x}{10}$$

$$\frac{6x}{6} = \frac{30}{6}$$

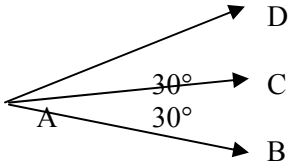
$$x = 5$$

(b) You currently have a triangular workbench in your workshop with the measurements shown in Diagram A below. You would like to build a larger workbench to replace the current one. You have already purchased a board that is 6 feet long and another board that is 8 feet long. How long would the third board need to be to complete the triangular workbench in the correct proportions?

Diagram A

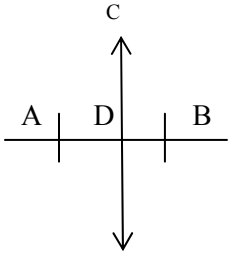


- Identifying angle bisectors, perpendicular bisectors, congruent angles, and congruent segments
  Examples: angle bisectors–



$\overrightarrow{AC}$ is an angle bisector of $\angle DAB$

perpendicular bisectors–



$\overrightarrow{DC}$ is a perpendicular bisector of $\overline{AB}$

Benchmark 13: Apply properties of similar polygons.

Example: Diana was given an assignment in drafting class to draw a replica of her school. The building is 15 meters high and 150 meters wide. Diana wants to draw a replica of the school that is 1 meter high. How many meters wide would the replica need to be?

Measurement

Algebra Content Standard 11: Solve problems algebraically that involve area and perimeter of a polygon, area and circumference of a circle, and volume and surface area of right circular cylinders or right rectangular prisms.

Benchmark 14: Apply appropriate formulas for perimeter, circumference, surface area, area, and volume in word problems.

Example: The carpentry class at the technical school is making circular picnic tables for the local park. If the radius of a picnic table is 2 feet, find the circumference of the table in order to calculate the number of feet of edging to go around the outside of the table.

Geometry Content Standard 16: Calculate surface areas and volumes of solid figures, including spheres, cones, and pyramids.

Benchmark 15: Determine surface area and volume of rectangular prisms, cylinders, and pyramids.

- Estimating surface area and volume
- Using appropriate units of measure to describe surface area and volume
- Determining surface area and volume by measuring with customary and metric units

  Example: The local oil company needs to have a canister built that will hold 500 cubic inches of oil. If the height of the canister needs to be 50 inches, what would the radius of the base of the canister need to be?

$$
\begin{array}{rcl}
V & = & \pi r^2 h \\
500 & = & \pi r^2(50) \\
500 & = & 3.14(50)(r^2) \\
\frac{500}{157} & = & \frac{157(r^2)}{157} \\
\sqrt{3.18} & = & \sqrt{r^2} \\
1.78 \text{ in.} & & r
\end{array}
$$

Data Analysis and Probability

Algebra Content Standard 14: Use a scatterplot and its line of best fit or a specific line graph to determine the relationship existing between two sets of data, including positive, negative, or no relationship.

Benchmark 16: Organize collected data, including representing the data graphically.

- Scatterplots

  Example: The brick masonry class did a study to determine the correlation between the number of students present in class each day and the number of bricks laid in class each day. The observations are recorded on the table below. Plot the points on a scatterplot and determine if the positive or negative correlation occurs.

Day	Number of Students Present	Number of Bricks Laid
1	6	45
2	8	60
3	10	82
4	9	62
5	7	56

- Line graphs

Algebra Content Standard 15: Estimate probabilities given data in lists or graphs.

Benchmark 17: Determine the theoretical probability of an event.

- Calculating the probability of complementary events and mutually exclusive events
  Example: Suppose a die numbered 1 – 6 is rolled once. What is the probability of rolling a 2?

  $$P = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$

  One is the number of favorable outcomes because there is only one number 2 on the die. Six is the number of possible outcomes because there are six numbers on the die. Therefore, P = 1/6.
  The complement to this outcome is 1 - P or 1- 1/6 = 5/6 = probability of rolling a number other than 2.

- Computing the probability of two independent events and two dependent events
  Examples: independent event–
  If the numbers from the example above are placed back in the bag after each trial, then the probability of the first trial will be 1/2. However, the probability of the second trial will also be 1/2 because the number of possible outcomes and number of favorable outcomes did not change. The probability of choosing two odd numbers during the first and second trials will then be P = 1/2 x 1/2 = 1/4.

  dependent event–
  Ten students in a classroom are assigned a number from 1 to 10. The numbers are placed in a bag. As the numbers are drawn, they are discarded.
  In the first trial, what is the probability that a student assigned an odd number will be chosen? (Answer: P = 5/10 = 1/2)
  Suppose an odd number was chosen in the first trial and removed. What will be the probability of choosing an odd number during the second trial? (Answer: 4/9)
  What is the probability of choosing two odd numbers during the first and second trials?
  P = 1/2 x 4/9 = 4/18 = 2/9 (Answer: 1/2 = probability of drawing an odd number in the first trial; 4/9 = probability of drawing an odd number in the second trial)

<u>Geometry Content Standard 18</u>: Construct with precision a circle graph to represent data from given tables or classroom experiments.

Benchmark 18: Construct circle graphs based on budget information.

- Determining the percentages of a budget (based upon a monthly net income) to be spent on each budgeted item
  - Housing
    - Banker's Rule
    - Renter's Rule
  - Utilities
  - Transportation
  - Food
  - Clothing
  - Insurance
  - Savings and/or investments
  - Medical expenses
  - Recreation
  - Education
  - Retirement

APPENDIX

Course Sequence Options

There are several options for mathematics course sequences for students who enter the AOD program in Grade 9. Some options for Grades 9 – 12 are listed below.

Student entering the AOD program in Grade 9 with a Grade 4 – Grade 5 level in mathematics:

Essential Skills Mathematics I
Essential Skills Mathematics II
Algebraic Explorations I
Algebraic Explorations II

Student entering the AOD program in Grade 9 with a Grade 5 – Grade 6 level in mathematics:

Essential Skills Mathematics II
Algebraic Explorations I
Algebraic Explorations II
Algebra IA or Algebra I

Student entering the AOD program in Grade 9 with a Grade 6 – Grade 7 level in mathematics:

Algebraic Explorations I
Algebraic Explorations II
Algebra IA or Algebra I
Algebra IB or Geometry

Student entering the AOD program in Grade 9 with a Grade 7 – Grade 8 level in mathematics:

Algebraic Explorations II
Algebra IA or Algebra I
Algebra IB or Geometry
Geometry or Algebraic Connections