CURRICULUM GUIDE
to the Alabama Course of Study

Mathematics
GRADES 1-8

Curriculum Guide
to the Alabama Course of Study: Mathematics
Grades 1-8

T a b l e   o f   C o n t e n t s

Acknowledgments

This document was developed by the July 2003 Mathematics Curriculum Guide Task Force. Task Force members included special and general education teachers and coordinators. In addition, parents of children with disabilities and representatives of parent advocacy groups reviewed the document.

Introduction

The *Curriculum Guide to the Alabama Course of Study: Mathematics* (Bulletin 2003, No. 42) is a companion document to the Grades 1 – 8 curriculum of the *Alabama Course of Study: Mathematics* (Bulletin 2003, Number 4). The *Alabama Course of Study: Mathematics* was developed by members of the Mathematics State Course of Study Committee and was adopted by the State Board of Education in February 2003. The *Alabama Course of Study: Mathematics* was developed in compliance with regulations of the *No Child Left Behind (NCLB) Act of 2001*. These regulations require (1) that content standards and state assessments have 100 percent alignment by 2004 in at least reading and mathematics and (2) that content not be repeated from grade to grade. These mathematics standards provide the basis for state assessments for purposes of state accountability. These standards can be accessed from the Alabama Department of Education web site at www.alsde.edu. On the home page, go to *Special Links*. Click on *Courses of Study, Mathematics*.

Educators are reminded that content standards indicate minimum content: what all students should know and be able to do by the end of each grade or course. Local systems may have additional instructional or achievement expectations, and they may provide instructional guidelines that include sequence, review, and/or remediation.

The *Curriculum Guide to the Alabama Course of Study: Mathematics* prepares students for study of grade-level content standards through the teaching of prerequisite and enabling skills necessary for learning each content standard. This allows students to work toward grade-level standards while working at individual ability levels. By identifying the prerequisites and enabling skills for each standard, teachers may plan instruction to close the achievement gap experienced by some students while still working toward the same standards set for all students. Some uses of the guide include, but are not limited to the following: (1) lesson planning, (2) Building-Based Student Support Team (BBSST) considerations, (3) Individual Educational Program (IEP) development, (4) collaborative teaching, (5) tutorials, (6) planning for instructional groupings, (7) parent information and conferences, (8) development of curriculum-based assessments, and (9) preparation for state assessments.

Organization of the Curriculum Guide

The organizational components of this guide include standards, instructional objectives, examples, and bullets. **Content standards** are statements that define what all students should know and be able to do at the conclusion of a course or grade. Content standards contain minimum required content and complete the phrase "*Students will.*"

Content standards for a grade level or course should be clearly written, reasonable, measurable, developmentally appropriate, and sufficiently rigorous to enable Alabama students to achieve at levels comparable to other students in the nation and the world. They should also provide proportional emphasis to the essential knowledge, skills, and processes of a given grade level or course.

Instructional objectives divide the standards into smaller instructional units that serve as foundational skills for the standards. Instructional objectives are useful in lesson planning, classroom instruction, and IEP development. Utilization of instructional objectives facilitates having all students working toward grade-level standards while also working at individual ability levels. Instructional objectives within this document are numbered according to grade level, content standard number, and the order in which the instructional objective is listed. The system for numbering **Objective 1. 2. 2.,** for example, is based upon the following:

grade level

content standard number

objective

Objective 1. 2. 2: Demonstrate addition and subtraction by joining or separating sets of objects.
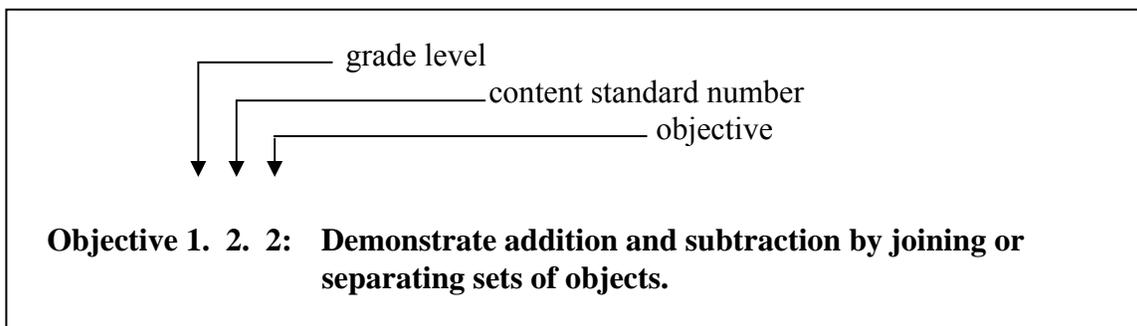
Examples clarify certain content standards and bullets, and/or their components. They are illustrative but not exhaustive. Examples are not part of the minimum required content.

Bullets denote additional related content required for instruction. Bulleted content is listed under a standard.

How Can Teachers Most Effectively Use This Document?

- Become familiar with the *Alabama Course of Study: Mathematics* (Bulletin 2003, No. 4).

- Review the supporting *Curriculum Guide to the Alabama Course of Study: Mathematics* (Bulletin 2003, No. 42) for Grades 1 – 8.

- Review the *Curriculum Guide to the Alabama Course of Study: Mathematics Prerequisites for Algebra I and Essential Skills for the Alabama Occupational Diploma* for Grades 9 – 12.

- Correlate standards and instructional objectives in the guide with the *Compendium Supplement for the Stanford Achievement Test, 10th Edition*.

- Correlate the standards and instructional objectives in the guide with the *Item Specifications for Mathematics* for the *Alabama High School Graduation Exam.*

- Use the guide and correlations for instructional planning.

- Teach all content specified in courses of study for each grade level or course.

- Emphasize the importance of vocabulary in all content areas.

- Develop curriculum-based assessments based on the standards.

- Make content relevant to real-life situations.

- Provide guided and independent practice.

- Plan and implement activities that address all learning styles: auditory, visual, kinesthetic, and tactile.

- Provide opportunities for cooperative and/or group learning.

- Include hands-on and other active learning experiences to increase student understanding.

Mathematics Standards and Instructional Objectives

First Grade

Number and Operations

Students will:

1. **Demonstrate concepts of number sense by counting forward and backward by ones, twos, fives, and tens up to 100; counting forward and backward from an initial number other than 1; and using multiple representations for a given number.**

   Objective 1.1.1: Count numbers in sequence by ones from 1 – 10.
   Objective 1.1.2: Identify a coin by name.
   Objective 1.1.3: Use vocabulary terms, including *more than, less than*, and *most* or *least,* to show comparison between sets with up to ten objects.

   Additional content to be taught:
   - Identifying position using the ordinal numbers 1st through 10th
   - Using vocabulary, including the terms *equal*, *all*, and *none*, to identify sets of objects
   - Recognizing that the quantity remains the same when the spatial arrangement changes
   - Determining the value of the digit in the ones place and the value of the digit in the tens place in a numeral
   - Determining the value of a number given the number of tens and ones
        Example: one ten and four ones = 14
   - Determining the value of a number that is 10 more or 10 less than a given number
   - Determining the monetary value of individual coins and sets of like coins up to $1.00

2. **Demonstrate conceptual understanding of addition and subtraction by telling number stories; joining, separating, and comparing sets of objects; and applying signs (+ and -) to the actions of joining and separating sets.**

   Objective 1.2.1:   Identify signs for addition and subtraction (+ and -).
   Objective 1.2.2:   Demonstrate addition and subtraction by joining or separating sets of objects.

   Additional content to be taught:
   - Solving simple word problems using a variety of strategies and distinguishing between relevant and irrelevant information
       Example: strategies—counting all, counting on, counting back
   - Solving problems requiring the addition and subtraction of one- or two-digit numerals without regrouping
   - Using three or more addends

3. **Demonstrate computational fluency of basic addition and subtraction facts by identifying sums to 10 and differences with minuends of 10 or less.**
       Example: giving an oral or written response to $3 + 2 = \underline{}$ or $\begin{array}{r} \mathbf{3} \\ \underline{+2} \end{array}$

   Objective 1.3.1:   Recognize numerals 1 – 10.
   Objective 1.3.2:   Identify "sum" as the answer to an addition problem and "difference" as the answer to a subtraction problem.

4. **Identify parts of a whole with two, three, or four equal parts.**

   Objective 1.4.1:   Recognize that an object can be divided into equal parts.

   Additional content to be taught:
   - Dividing an object into equal parts

Algebra

5. **Create repeating patterns.**

   Objective 1.5.1:   Replicate patterns using concrete objects.

   Additional content to be taught:
   - Describing characteristics of patterns
   - Extending patterns including number patterns
   - Identifying patterns in the environment

6. **Solve problems using the identity and commutative properties of addition.**
   Examples: $6 + 3 =$ ___ $+ 6$ (identity property),
   $6 +$ ___ $= 6$ and ___ $+ 6 = 6$ (commutative property)

   Objective 1.6.1: Count numbers in sequence by ones from $1 - 20$.
   Objective 1.6.2: Recognize numerals $0 - 9$.
   Objective 1.6.3: Relate numerals $0 - 9$ to numbers of objects in a set.

7. **Demonstrate relationships between operations.**
   Example: addition and subtraction fact families—
   $5 + 2 = 7$ $7 - 2 = 5$
   $2 + 5 = 7$ $7 - 5 = 2$

   Objective 1.7.1: Count numbers from $0 - 20$ in sequence by ones forward and backward.
   Objective 1.7.2: Identify $+$, $-$, and $=$ as the symbols for addition, subtraction, and equality.

Geometry

8. **Differentiate among plane shapes, including circles, squares, rectangles, and triangles.**

   Objective 1.8.1: Create combinations of rectangles, squares, circles, and triangles using shapes or drawings.
   Example:
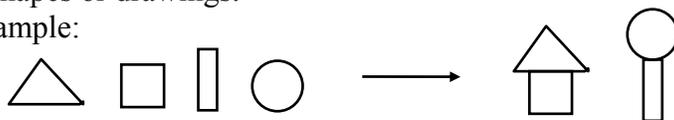
   Objective 1.8.2: Identify rectangles, squares, circles, and triangles.

   Additional content to be taught:
   - Describing similarities and differences between plane and solid shapes
     Examples: round, flat, curved, straight
   - Transferring shape combinations from one representation (dimension) to another
     Examples: making a particular grouping of blocks by using a drawing of the grouping, making a drawing of a specific arrangement of blocks
   - Recognizing real-life examples of line symmetry
     Example: recognizing a line of symmetry in a piece of folded paper or an orange cut in half
   - Changing the position of objects or shapes by sliding (translation) and turning (rotation)
   - Combining shapes to fill in the area of a given shape
     Example: covering a rectangle with two triangles

9. **Identify solid shapes in the environment, including cubes, rectangular prisms, cones, spheres, and cylinders.**

   Objective 1.9.1:   Recognize plane shapes in the environment.
   Examples:

   (a) yield sign as a triangle   (b) door/desktop as a rectangle

Measurement

10. **Compare objects according to length, weight, and capacity.**
    Examples: (a) length
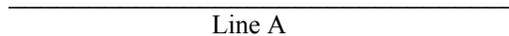
    _____
    Line A
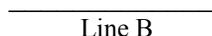
    _____
    Line B
    Which line is shorter: line A or line B?

    (b) weight
    Which weighs more: an elephant or a monkey?

    (c) capacity

    Which jar holds more: the one on the left or the one on the right?

    Objective 1.10.1: Use vocabulary associated with length, height, volume, and weight to compare objects.
    Examples: longer than, as long as, shorter than, as short as, taller than, as tall as, holds more, as heavy as

    Additional content to be taught:
    - Measuring the length of objects using a variety of nonstandard units
      Examples: using objects of unequal length—finding number of pencils needed to measure length of desk,
      using objects of equal length—comparing number of equally sized paper clips needed to measure length of desk
    - Ordering according to attributes

11. **Identify the hour using analog and digital clocks.**

   Objective 1.11.1: Recognize numerals 0 – 12.

   Additional content to be taught:
   - Identifying the half hour using analog and digital clocks

12. **Locate days, dates, and months on a calendar.**
   Examples: locating the third Thursday of the month on a calendar; recognizing that today is Tuesday, January 24

   Objective 1.12.1: Recognize numerals 0 – 31.
   Objective 1.12.2: Use vocabulary associated with the measurement of time, including words related to clocks and calendars.

   Additional content to be taught:
   - Using vocabulary associated with a calendar
       Example: using the words yesterday, today, tomorrow, day before, day after

Data Analysis and Probability

13. **Organize objects or information into predetermined and labeled data displays, including pictographs, tally charts, bar graphs, or double-loop Venn diagrams.**

   Objective 1.13.1:   Recognize different types of data displays.

   Additional content to be taught:
   - Generating simple questions for data collection
       Example: "Do you like chocolate ice cream?"
   - Creating displays with appropriate labels
       Example:
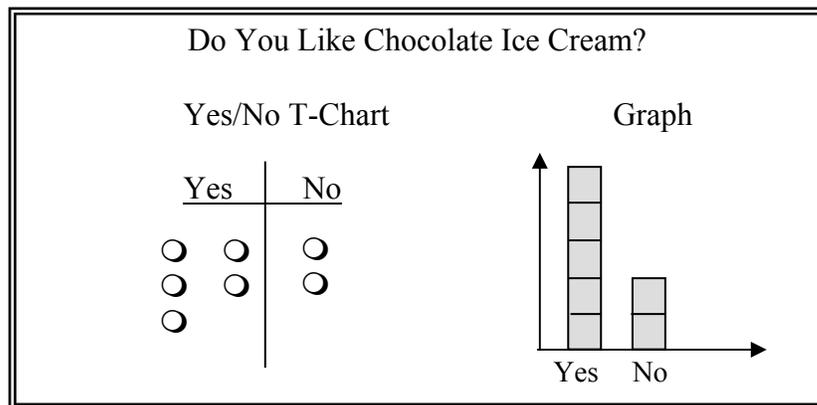
Second Grade

Number and Operations

Students will:

1. **Demonstrate concepts of number sense by using multiple representations of whole numbers up to 1000, counting forward and backward by threes from a given number, identifying a number that is 100 more or 100 less than a given number, and differentiating between odd and even numbers.**

   Objective 2.1.1: Determine the value of the digit in the ones place and the value of the digit in the tens place.
   Objective 2.1.2: Count forward and backward by ones and twos.
   Objective 2.1.3: Determine the value of a number that is 10 more or 10 less than a given number.

   Additional content to be taught:
   - Identifying position using ordinal numbers to 100th
   - Determining the value of a digit in the ones, tens, hundreds, and thousands place
   - Determining the value of a number expressed in expanded notation
       Example: $700 + 70 + 3 = 773$

2. **Apply the operations of addition and subtraction to solve problems involving two-digit numerals, using multiple strategies with and without regrouping.**
    Example: using concrete objects, mental calculations, or paper-and-pencil activities

    Objective 2.2.1:   Demonstrate strategies, using objects, to solve addition and subtraction problems.
        Example: using place value manipulatives to demonstrate $34 + 7$ (regrouping strategy)
    Objective 2.2.2:   Solve problems requiring the addition and subtraction of one-digit or two-digit numerals without regrouping.

    Additional content to be taught:
    - Demonstrating computational fluency for basic addition and subtraction facts with sums through 18 and differences with minuends through 18, using horizontal and vertical forms
    - Interpreting multiplication as repeated addition and division as equal groupings
        Examples: $3 \times 5 = 5 + 5 + 5 =$

        $20 \div 4 = 5$
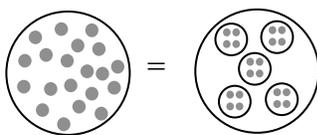
    - Solving multistep addition and subtraction problems originating from real-life experiences
        Example: There were 5 students on the bus after the first stop. Three students got on at the second stop. The bus made one more stop before arriving at school. When the bus arrived at school, 18 students got off. How many students got on at the last stop?
    - Justifying the strategy used to solve addition and subtraction problems
    - Using an estimate to determine if an answer is reasonable

3. **Label equal parts of a whole using $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$.**

    Objective 2.3.1:   Recognize that a whole object can be divided into equal parts.
        Example: cutting a pizza into halves, thirds, and fourths

4. **Determine the monetary value of sets of coins and bills up to $2.00.**

   Objective 2.4.1:   Recognize denominations of coins and currency.
   Objective 2.4.2:   Determine the monetary value of individual coins and sets of like coins up to $1.00.

   Additional content to be taught:
   - Exchanging coins of equivalent value
   - Applying monetary symbols, including dollar ($), cent (¢), and decimal point (.)
   - Recognizing the decimal numbers .10, .25, .50, and .75 as related to money

Algebra

5. **Create growing patterns.**
   Example: ▢, ▢▢, ▢▢▢; a b, a a b, a a a b

   Objective 2.5.1:   Replicate patterns by using concrete objects.
   Objective 2.5.2:   Determine if a sequence of objects forms a pattern.
   Objective 2.5.3:   Complete a repeating pattern.
   Example: ▢ ▢ △ ▢ ▢ △

   Draw the next four objects in the above pattern.

6. **Solve problems using the associative property of addition.**

   Objective 2.6.1:   Demonstrate basic addition and subtraction facts by identifying sums to 10 and differences with minuends of 10 or less.
   Objective 2.6.2:   Identify an example of the associative property of addition.
   Example: Which of the following problems is an example of the associative property of addition? (a) $3 + (5 + 2) = (3 + 5) + 2$
   or (b) $3 + 5 = 5 + 3$ (Answer: problem a)

7. **Describe change over time in observable (qualitative) and measurable (quantitative) terms.**
   Examples: recognizing that a plant grew taller (qualitative, requiring observation); recognizing that a plant grew three inches (quantitative, requiring measurement)

   Objective 2.7.1:   Compare objects according to length, weight, and capacity.

Geometry

8. **Describe attributes of two-dimensional (plane) and three-dimensional (solid) figures using the terms *side*, *surface*, *edge*, *vertex*, and *angle*.**

   Objective 2.8.1: Identify side, surface, edge, vertex, or angle of a two- or three-dimensional figure.
   Objective 2.8.2: Identify solid (three-dimensional) shapes, including cubes, rectangular prisms, cones, spheres, and cylinders.

   Additional content to be taught:
   - Identifying quadrilaterals, pentagons, hexagons, or octagons
   - Identifying line symmetry in plane geometric figures
   - Creating designs that exhibit line symmetry
   - Recognizing the results of changing the position (transformation) of objects or shapes by sliding (translation), turning (rotation), or flipping (reflection)
       Examples: sliding (vertically)   turning   flipping (horizontally)

9. **Describe the route from one location to another by applying concepts of direction and distance.**
       Examples: direction—left, right, north, south, east, west;
                 distance (nonstandard)—twenty-five steps from the library;
                 distance (standard)—ten feet from the walkway

   Objective 2.9.1: Identify left, right, up, and down.
   Objective 2.9.2: Locate north, south, east, and west on a map.

   Additional content to be taught:
   - Following multistep directions to locate objects
   - Reading maps of the school environment
       Example: using a school map to tell how to get from the classroom to the office
   - Using grids for movement between points
       Example: moving from the house ( ) to the tree ( ) by moving two down and five over on the grid
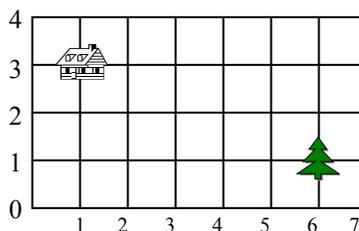
Measurement

10. **Measure length in customary units, including inches, feet, and yards.**

    Objective 2.10.1: Identify inch, foot, and yard on various tools of measurement.
    Objective 2.10.2: Measure the length of objects using a variety of nonstandard units.

    Additional content to be taught:
    - Using metric units
    - Using appropriate tools, including rulers, yard sticks, meter sticks, or tape measures

11. **Estimate weight and capacity by making comparisons with familiar objects.**

    Objective 2.11.1: Use the terms *more than* and *less than* to compare weight or capacity of objects.
    > Examples: a desk weighing more than a pencil, a cup holding less than a bucket
    Objective 2.11.2: Compare objects according to weight and capacity.

12. **Tell time to the minute using analog and digital clocks.**

    Objective 2.12.1: Recognize numerals 0 – 60.
    Objective 2.12.2: Demonstrate concepts of number sense by counting forward and backward by ones, fives, and tens up to 60.
    Objective 2.12.3: Recognize time on an analog or digital clock to the quarter hour.
    Objective 2.12.4: Use vocabulary associated with the measurement of time, including words related to clocks and calendars.
    > Examples: before, after, first, last, hours, days, weeks, months

Data Analysis and Probability

13. **Create displays, including appropriate labels, for a given set of data using pictographs, tally charts, bar graphs, or single- or double-loop Venn diagrams.**

    Objective 2.13.1: Organize information or objects into predetermined and labeled data displays.

    Additional content to be taught:
    - Interpreting graphic displays

14. **Determine if one event related to everyday life is more likely or less likely to occur than another event.**

   Example: determining if it is more likely to rain or snow on July 4th in
                   Alabama

Objective 2.14.1: Explain the meaning of the terms *more likely* and *less likely*.

Third Grade

Number and Operations

Students will:

1. **Demonstrate number sense by comparing, ordering, and expanding whole numbers through 9999.**

   Objective 3.1.1:   Compare numbers less than 1000.
   Objective 3.1.2:   Order numbers less than 1000.

   Additional content to be taught:
   - Comparing numbers using the symbols $>$, $<$, $=$, and $\neq$
   - Identifying the place value of any digit within a four-digit number
   - Writing a four-digit number in words and locating it on a number line
   - Determining the value of a number written in expanded notation to the ten-thousands place
       Example: $10,000 + 3,000 + 400 + 20 + 1 = 13,421$
   - Rounding whole numbers to the nearest ten and hundred and money values to the nearest dollar

2. **Solve addition and subtraction problems, including word problems, involving two- and three-digit numbers with and without regrouping.**

   Objective 3.2.1:   Apply regrouping strategies to addition and subtraction problems.
   Objective 3.2.2:   Solve simple word problems involving single-digit numbers.

   Additional content to be taught:
   - Estimating sums and differences by using compatible numbers, front-end estimation, and rounding
       Examples: compatible numbers— $24 + 26 = 25 + 25$

       front-end estimation— $\begin{array}{r} 72 \\ -36 \end{array}$ is approximately $\begin{array}{r} 70 \\ -30 \end{array}$

       rounding— $\begin{array}{r} 172 \\ +369 \end{array}$ is approximately $\begin{array}{r} 200 \\ +400 \end{array}$

   - Demonstrating computational fluency in addition and subtraction

3. **Multiply whole numbers with and without regrouping using single-digit multipliers.**

   Objective 3.3.1: Recognize multiplication as repeated addition of equal groupings.
   Objective 3.3.2: Apply the operation of addition to solve problems involving two-digit numbers using multiple strategies with regrouping.

   Additional content to be taught:
   - Applying concepts of multiplication through the use of manipulatives, number stories, arrays, repeated addition, or problem situations
   - Applying basic multiplication facts through 9 x 9 by using manipulatives, solving problems, and writing number stories
   - Recognizing properties of multiplication

4. **Divide whole numbers using two-digit dividends and one-digit divisors.**

   Objective 3.4.1: Demonstrate division as equal groupings.
   Objective 3.4.2: Apply the operation of subtraction to solve problems involving two-digit numbers using multiple strategies with regrouping.

   Additional content to be taught:
   - Recognizing division as repeated subtraction

5. **Model equivalent fractions with concrete objects or pictorial representations.**

   Objective 3.5.1: Identify parts of a whole with two, three, or four equal parts.
   Objective 3.5.2: Label equal parts of a whole using $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$.

6. **Use coins to make change up to $1.00.**

   Objective 3.6.1: Exchange coins of equivalent value.
         Example: two dimes and one nickel equaling one quarter
   Objective 3.6.2: Determine the monetary value of sets of coins up to $1.00.

   Additional content to be taught:
   - Determining monetary values of sets of unlike coins and bills up to $5.00

Algebra

7. **Complete a given numeric or geometric pattern.**
    Examples: geometric pattern—△ ☐ △ △ ☐ △ △ ___ ☐;
    numeric pattern—20, 27, 34, 41, ____

    Objective 3.7.1: Create repeating patterns.
    Objective 3.7.2: Create growing patterns.

Geometry

8. **Identify geometric representations for points, lines, perpendicular lines, parallel lines, angles, and rays.**

    Objective 3.8.1: Describe attributes of plane (two-dimensional) figures using the terms *side, vertex,* and *angle.*

    Additional content to be taught:
    - Recognizing real-life examples of points, lines, perpendicular lines, and parallel lines
    - Drawing points, lines, and perpendicular lines

9. **Specify locations of a coordinate grid by using horizontal and vertical movements.**

    Objective 3.9.1: Demonstrate movement horizontally and/or vertically on a grid from 0 to a given number.
    Objective 3.9.2: Demonstrate movement horizontally or vertically on a grid from a point other than 0 to a given point.
    Example: moving from point **A** to point **B** by moving two down and five over on the grid
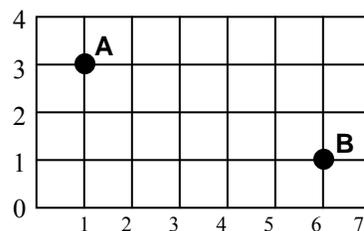
Measurement

10. **Measure length in metric units.**

   Objective 3.10.1: Identify numeric value of metric prefixes.

   Examples: milli $- \dfrac{1}{1,000}$, centi $- \dfrac{1}{100}$, deci $- \dfrac{1}{10}$, meter $- 1$

   Objective 3.10.2: Utilize appropriate tools including rulers, tape measures, and/or meter sticks, to measure given objects.

   Objective 3.10.3: Identify metric measurements of millimeter, centimeter, decimeter, and meter on measurement tools.

11. **Determine elapsed time to the day with calendars and to the hour with a clock.**

   Objective 3.11.1: Tell time to the minute using analog and digital clocks.

   Objective 3.11.2: Locate days, dates, and months on a calendar.

   Objective 3.11.3: Apply the operations of addition and subtraction to solve problems involving two-digit numbers with and without regrouping.

   Additional content to be taught:
   - Calculating elapsed time to the minute within the same hour
   - Applying vocabulary associated with time using *a.m., p.m., noon,* or *midnight*

Data Analysis and Probability

12. **Recognize data as either categorical or numerical.**
   Examples: categorical—gender, race, languages spoken, genre;
   numerical—age, height, weight

   Objective 3.12.1: Interpret graphic displays.

   Additional content to be taught:
   - Comparing related data sets

13. **Determine the likelihood of different outcomes in a simple experiment.**
   Example: determining that the spinner is least likely to land on red in this diagram

   Objective 3.13.1: Determine if one event related to everyday life is more likely or less likely to occur than another event.

Fourth Grade

Number and Operations

Students will:

1. **Demonstrate number sense by comparing and ordering decimals to hundredths and whole numbers to 999,999.**

   Objective 4.1.1: Identify the place value of any digit within a four-digit number and in a decimal to hundredths.
   Objective 4.1.2: Compare numbers using the symbols >, <, =, and ≠.
   Objective 4.1.3: Write a number in expanded notation through the ten thousands.
   Example: $14,682 = 10,000 + 4,000 + 600 + 80 + 2$

   Additional content to be taught:
   - Identifying a number when given a pictorial representation of tenths and hundredths or groups of ones, tens, hundreds, and thousands
   - Writing a number in expanded notation through the hundred-thousands
     Example: $914,682 = 900,000 + 10,000 + 4,000 + 600 + 80 + 2$
   - Determining the place value of a digit in a whole number through the hundred-thousands and in a decimal to the hundredths

2. **Write money amounts in words and dollar-and-cent notation.**

   Objective 4.2.1: Write a four-digit number in words.
   Example: 1,433—one thousand, four hundred thirty-three
   Objective 4.2.2: Apply monetary symbols, including dollar ($), cent (¢), and decimal point (.).

   Additional content to be taught:
   - Identifying equivalent units of money

3. **Rename improper fractions as mixed numbers and mixed numbers as improper fractions.**

   Objective 4.3.1: Recognize equivalent forms of commonly used fractions.
   Objective 4.3.2: Apply basic multiplication and division facts to solve problems.

   Additional content to be taught:
   - Using a number line to simplify, compare, and order fractions and mixed numbers
   - Writing equivalent forms of fractions

4. **Demonstrate addition and subtraction of fractions with common denominators.**

    Objective 4.4.1: Demonstrate computational fluency in addition and subtraction.
    Objective 4.4.2: Divide whole numbers using two-digit dividends and one-digit divisors.

5. **Round whole numbers to the nearest ten, hundred, or thousand and decimals to the nearest tenth.**
    Examples: 17 rounds to 20, 668 rounds to 700, 8,820 rounds to 9,000, 0.17 rounds to 0.20

    Objective 4.5.1: Identify the place value of any digit in a four-digit number.
    Objective 4.5.2: Identify place values of decimals to the nearest hundredth.

6. **Solve problems, including word problems, that involve addition and subtraction of four-digit numbers with and without regrouping.**

    Objective 4.6.1: Apply regrouping strategies to addition and subtraction problems.

    | | Examples: with regrouping $-1,723$ | without regrouping $-2,354$ |
    | | $-\ 948$ | $-1,231$ |
    | | 775 | $1,123$ |

    Objective 4.6.2: Use estimation strategies to determine if an answer is reasonable.
    Objective 4.6.3: Recognize "product" as the answer to a multiplication problem and "quotient" as the answer to a division problem.
    Objective 4.6.4: Solve addition and subtraction problems, including word problems, involving two- and three-digit numbers with and without regrouping.

    Additional content to be taught:
    *   Estimating sums and differences of whole numbers by using appropriate strategies such as rounding, front-end estimation, and compatible numbers
    *   Adding and subtracting decimals and money amounts

7. **Solve problems, including word problems, involving the basic operations of multiplication and division on whole numbers through two-digit multipliers and one-digit divisors.**

    Objective 4.7.1: Solve addition and subtraction problems, including word problems, involving two- and three-digit numbers with and without regrouping.

    Additional content to be taught:
    *   Estimating products and quotients of whole numbers by using appropriate strategies such as rounding, front-end estimation, and compatible numbers
    *   Identifying information needed to determine the appropriate operation to solve a problem
    *   Demonstrating computational fluency in multiplication and division fact families through 12

8. **Recognize equivalent forms of commonly used fractions and decimals.**

   Examples: $\frac{1}{4}$ = .25, $\frac{1}{4}$ of a dollar = \$.25 (25 cents)

   Objective 4.8.1:   Model equivalent fractions with concrete objects or pictorial representations.

Algebra

9. **Write number sentences for word problems that involve multiplication or division.**

   Example: Johnny has a scrapbook with 5 pages. Each page has 4 pictures.
   How many pictures are in the book? (Answer: 5 x 4 = 20 pictures.)

   Objective 4.9.1:   Apply basic multiplication and division facts through 9 x 9 by solving problems and writing number stories.

10. **Complete addition and subtraction number sentences with a missing addend or subtrahend.**

    Objective 4.10.1: Find the sum or difference using whole numbers from 0 – 99.

Geometry

11. **Identify triangles, quadrilaterals, pentagons, hexagons, or octagons based on the number of sides, angles, and vertices.**

    Objective 4.11.1   Recognize the results of changing the position (transformation) of objects or shapes by sliding (translation), turning (rotation), or flipping (reflection).
    Objective 4.11.2: Define side, angle, and vertices in relation to two-dimensional figures.

    Additional content to be taught:
    - Demonstrating slides (translations), flips (reflections), and turns (rotations) using triangles, quadrilaterals, pentagons, hexagons, or octagons
    - Drawing lines of symmetry in triangles, quadrilaterals, pentagons, hexagons, or octagons

12. **Find locations on a map or grid using ordered pairs.**

   Objective 4.12.1: Describe the route from one location to another by applying
      concepts of direction and distance.
   Objective 4.12.2: Specify locations on a coordinate grid by using horizontal and
      vertical movements.

Measurement

13. **Calculate elapsed time in hours and minutes.**

   Objective 4.13.1: Tell time to the minute using analog and digital clocks.
   Objective 4.13.2: Calculate elapsed time to the minute within the same hour.
   Objective 4.13.3: Apply regrouping principals to time concepts.
         Example: borrowing hours to make minutes

14. **Measure length, width, weight, and capacity, using metric and customary units,
    and temperature in degrees Fahrenheit and degrees Celsius.**

   Objective 4.14.1: Identify appropriate tools for measuring length, width, and
      capacity, including rulers, yard sticks, meter sticks, tape measures, measuring
      cups, and scales.
   Objective 4.14.2: Select the appropriate unit of measure, metric or customary, to
      determine measurements.

   Additional content to be taught:
   - Estimating perimeter and area of irregular shapes using unit squares and grid
     paper
   - Estimating area using unit squares

Data Analysis and Probability

15. **Represent categorical data using tables and graphs, including bar graphs, line
    graphs, and line plots.**

   Objective 4.15.1: Identify categorical data.
   Objective 4.15.2: Create data displays for a given set of data using pictographs and
      bar graphs.

   Additional content to be taught:
   - Collecting data using observations, surveys, or experiments
   - Creating tally charts to represent data collected from real-life situations

16. **Determine if outcomes of simple events are likely, unlikely, certain, equally likely, or impossible.**

      Example: determining if it is likely, unlikely, certain, equally likely, or impossible for it to snow on July 4th in Alabama

      Objective 4.16.1: Define the terms *likely*, *unlikely*, *certain*, *equally likely*, and *impossible* in the context of probability.

17. **Represent numerical data using tables and graphs, including bar graphs and line graphs.**

      Objective 4.17.1: Identify numerical data.
      Objective 4.17.2: Create displays for a given set of data using bar graphs.

Fifth Grade

Number and Operations

Students will:

1. **Demonstrate number sense by comparing, ordering, rounding, and expanding whole numbers through millions and decimals to thousandths.**

   Objective 5.1.1: Identify place value from hundred thousands to hundredths.
   Example: 789402.6**5**4—**5** is in the hundredths place
   Objective 5.1.2: Round whole numbers to the nearest hundred thousands and decimals to the nearest hundredths.
   Examples: 199,999 rounds to 200,000,
   6.754 rounds to 6.75
   Objective 5.1.3: Compare numbers using the symbols >, <, =, and ≠ for whole numbers through the hundred thousands and decimals to the hundredths.

   Additional content to be taught:
   - Relating percents to parts out of 100 by using equivalent fractions and decimals
   - Determining the value of a digit to thousandths

2. **Solve problems involving basic operations on whole numbers, including addition and subtraction of seven-digit numbers, multiplication with two-digit multipliers, and division with two-digit divisors.**

   Objective 5.2.1: Solve problems that involve addition and subtraction of four-digit numbers with regrouping.
   Objective 5.2.2: Demonstrate computational fluency in multiplication and division fact families through 12.

   Additional content to be taught:
   - Estimating products and quotients
   - Determining divisibility by 2, 3, 4, 5, 6, 9, and 10
   - Demonstrating computational fluency with addition, subtraction, multiplication, and division of whole numbers

3. **Solve word problems that involve decimals, fractions, or money.**

   Objective 5.3.1: Recognize vocabulary associated with mathematical operations.
   Examples: difference being the result of subtraction,
   total or sum being the result of addition,
   product being the result of multiplication
   Objective 5.3.2: Demonstrate addition and subtraction of fractions with common denominators.
   Objective 5.3.3: Write number sentences for word problems that involve multiplication and division.
   Objective 5.3.4: Write money amounts in words and dollar-and-cent notation.

   Additional content to be taught:
   - Solving word problems involving elapsed time

4. **Determine the sum and difference of fractions with common and uncommon denominators.**

   Objective 5.4.1: Write equivalent forms of fractions.
   Objective 5.4.2: Demonstrate addition and subtraction of fractions with common denominators.

   Additional content to be taught:
   - Changing mixed numbers to improper fractions
   - Solving problems involving addition and subtraction of fractions with common and uncommon denominators
   - Using least common multiples
   - Estimating sums and differences of fractions

5. **Identify numbers less than zero by extending the number line.**
   Example: identifying negative temperatures (below 0°) on a thermometer

   Objective 5.5.1: Recognize whole numbers on a number line.

Algebra

6. **Demonstrate the commutative, associative, and identity properties of addition and multiplication of whole numbers.**

   Objective 5.6.1: Complete addition and multiplication number sentences with a missing addend or multiplier/multiplicand.
   Objective 5.6.2: Solve problems involving basic operations of addition and multiplication on whole numbers through four-digit addends and two-digit multipliers.

7. **Write a number sentence for a problem expressed in words.**

   Objective 5.7.1: Recognize mathematical terms and corresponding symbols.
   Examples: equals =, plus +, minus -, greater than >, less than <, not equal to ≠, parentheses ( )

   Objective 5.7.2: Write number sentences for word problems that involve multiplication or division.

Geometry

8. **Identify regular polygons and congruent polygons.**

   Objective 5.8.1: Identify triangles, quadrilaterals, pentagons, hexagons, or octagons based on the number of sides, angles, and vertices.

   Additional content to be taught:
   - Identifying angles as right, obtuse, acute, or straight
   - Classifying triangles as equilateral, isosceles, or scalene
   - Identifying figures that have rotational symmetry
   - Predicting the results of a flip (reflection), turn (rotation), or slide (translation)

9. **Identify components of the Cartesian plane, including the x-axis, y-axis, origin, and quadrants.**

   Objective 5.9.1: Find locations on a map or grid using ordered pairs.
   Objective 5.9.2: Specify locations on a coordinate grid by using horizontal and vertical movements.

10. **Identify the center, radius, and diameter of a circle.**

    Objective 5.10.1: Draw points, lines, and perpendicular lines.
    Objective 5.10.2: Identify geometric representations for points and lines.

Measurement

11. **Estimate perimeter and area of irregular shapes using unit squares and grid paper.**

    Objective 5.11.1: Estimate area of triangles, squares, and rectangles using unit squares.

12. **Calculate the perimeter of rectangles from measured dimensions.**

    Objective 5.12.1: Measure length and width using metric and customary units.

13. **Convert a larger unit of measurement to a smaller unit of measurement within the same system (customary or metric).**
    Examples: 4 cups = 32 fluid ounces, 2 meters = 200 centimeters,
                    2 miles = 10,560 feet

    Objective 5.13.1: Solve problems involving the basic operations of multiplication and division on whole numbers through two-digit multipliers and one-digit divisors.

Data Analysis and Probability

14. **Analyze data collected from a survey or experiment to distinguish between what the data show and what might account for the results.**

    Objective 5.14.1: Collect data using observations, surveys, or experiments.
    Objective 5.14.2: Represent numerical data using tables and graphs, including bar graphs and line graphs.

    Additional content to be taught:
    - Evaluating different representations of the same data to determine how well each representation shows important aspects of the data
    - Using given measures of central tendency (mean, median, and mode) to analyze data

15. **Use common fractions to represent the probability of events that are neither certain nor impossible.**
    Example: finding the probability of stopping on a vowel when using a spinner with three vowels and five consonants

    Objective 5.15.1: Recognize equivalent forms of commonly used fractions.
    Objective 5.15.2: Simplify fractions to lowest terms.

Sixth Grade

Number and Operations

Students will:

1. **Demonstrate computational fluency with addition, subtraction, multiplication, and division of decimals and fractions.**

   Objective 6.1.1: Determine the sum and difference of fractions with common and uncommon denominators.
   Objective 6.1.2: Rename improper fractions as mixed numbers and mixed numbers as improper fractions.
   Objective 6.1.3: Recognize equivalent forms of commonly used fractions and decimals.

   Additional content to be taught:
   - Comparing rational numbers written as fractions, decimals, mixed numbers, and percents
   - Converting fractions and mixed numbers to decimals and percents
   - Converting terminating decimals and percents to fractions and mixed numbers
   - Writing decimal numbers in expanded notation

     Example: $52.37 = 50 + 2 + \dfrac{3}{10} + \dfrac{7}{100}$

   - Using prime factorizations
   - Identifying prime and composite numbers
   - Using greatest common factor (GCF) to simplify fractions
   - Formulating algorithms using basic operations on fractions and decimals
     Example: determining a systematic set of steps that can be used to divide fractions
   - Applying the distributive property to compute with fractions and decimals
     Example: $4 \times (8\frac{1}{2}) = (4 \times 8) + (4 \times \frac{1}{2}) = 32 + 2 = 34$
   - Using least common multiple (LCM) to add and subtract fractions with unlike denominators

2. **Solve problems involving decimals, percents, fractions, and proportions.**
      Example: determining the sale price of a pair of jeans that regularly sells for
                      $25 to be $22.50 if they are on sale at 10 percent off the regular price

Objective 6.2.1:   Solve problems involving the addition, subtraction,
multiplication, and division of decimals and fractions.
Objective 6.2.2:   Rename improper fractions as mixed numbers and mixed
numbers as improper fractions.

Additional content to be taught:
- Estimating with fractions and decimals

Algebra

3. **Solve problems using numeric and geometric patterns.**
      Examples: (a)   predicting the cost of using a cell phone for 60 minutes when
                        given a bar graph

Cost of Using a Cell Phone
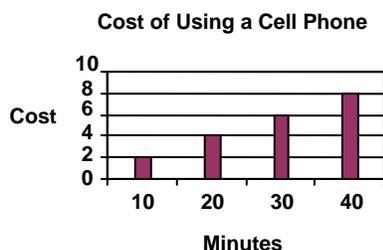
       (b)   continuing a pattern for the 5th and 6th numbers when given the
first four numbers in the pattern

1st   2nd   3rd   4th
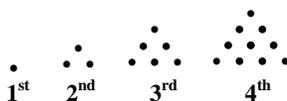
Objective 6.3.1:   Represent numerical data using tables and graphs, including bar
graphs and line graphs.

Additional content to be taught:
- Determining a verbal rule for a function given the input and output
     Example:

Number of Hours Worked	1	2	3	4	5
Number of Dollars Earned	5	10	15	20	25

(Rule: number of dollars earned is five times number of hours worked)

Geometry

4. **Identify two-dimensional and three-dimensional figures based on attributes, properties, and component parts.**

   Objective 6.4.1: Identify figures that have rotational symmetry.
   Objective 6.4.2: Identify triangles, quadrilaterals, pentagons, hexagons, or octagons based on the number of sides, angles, and vertices.

   Additional content to be taught:
   - Classifying quadrilaterals based on their attributes
   - Identifying line and rotational symmetries of polygons
   - Classifying triangles as right, obtuse, or acute

5. **Plot coordinates on grids, graphs, and maps.**

   Objective 6.5.1: Identify components of the Cartesian plane, including the x-axis, y-axis, origin, and quadrants.
   Objective 6.5.2: Identify numbers less than zero by extending the number line.
   Objective 6.5.3: Find locations on a map or grid using ordered pairs.

   Additional content to be taught:
   - Identifying the coordinates of a point on the Cartesian plane
   - Comparing parallel and perpendicular lines

Measurement

6. **Classify angles as acute, obtuse, right, or straight.**

   Objective 6.6.1: Identify geometric representations for points, lines, perpendicular lines, parallel lines, angles, and rays.

   Additional content to be taught:
   - Estimating angle measures using 45 degrees, 90 degrees, 180 degrees, 270 degrees, or 360 degrees as referents
   - Measuring angles

7. **Solve problems involving perimeter and area of parallelograms and rectangles.**

   Objective 6.7.1: Estimate perimeter and area of irregular shapes using unit squares and grid paper.
   Objective 6.7.2: Calculate the perimeter of rectangles from measured dimensions.
   Objective 6.7.3: Measure length and width using metric and customary units.

   Additional content to be taught:
   - Estimating perimeter and area
   - Developing formulas to determine perimeter and area of parallelograms and rectangles

8. **Determine the distance between two points on a scale drawing or a map using proportional reasoning.**

   Objective 6.8.1: Plot coordinates on grids, graphs, and maps.
   Objective 6.8.2: Solve problems involving decimals, percents, fractions, and proportions.
   Objective 6.8.3: Determine the sum and difference of fractions with common and uncommon denominators.

   Additional content to be taught:
   - Using different forms of notation to symbolize ratios and rates

9. **Convert units of length, weight, or capacity within the same system (customary or metric).**

   Objective 6.9.1: Convert a larger unit of measurement to a smaller unit of measurement within the same system, customary or metric.
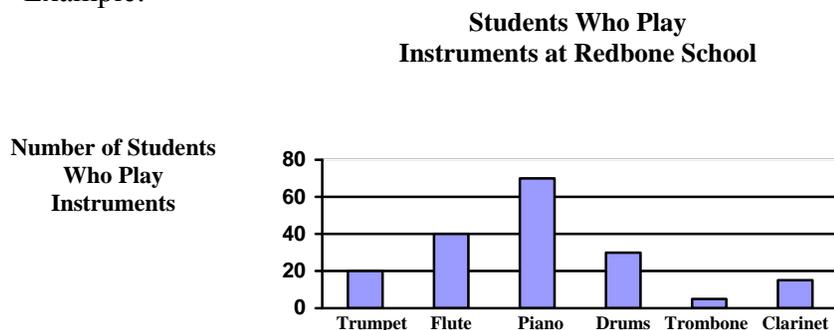   **Objective 6.9.2:** Measure length, width, weight, and capacity, using metric and customary units.
   Objective 6.9.3: Rename improper fractions as mixed numbers and mixed numbers as improper fractions.

Data Analysis and Probability

10. **Interpret information from bar graphs, line graphs, and circle graphs.**
    Example:

    Students Who Play
    Instruments at Redbone School

    Number of Students
    Who Play
    Instruments

    Possible conclusions—Twice as many students
    play flute as trumpet. Redbone needs
    more trombone players.

Objective 6.10.1: Analyze data collected from a survey or experiment to distinguish between what the data show and what might account for the results.

Objective 6.10.2: Represent numerical data using tables and graphs, including bar graphs and line graphs.

Objective 6.10.3: Represent categorical data using tables and graphs, including bar graphs, line graphs, and line plots.

11. **Find the probability of a simple event.**

Objective 6.11.1: Use common fractions to represent the probability of events that are neither certain nor impossible.

Objective 6.11.2: Determine if outcomes of simple events are likely, unlikely, certain, equally likely, or impossible.

Additional content to be taught:
- Expressing probabilities as ratios, percents, and decimals

Seventh Grade

Number and Operations

Students will:

1. **Demonstrate computational fluency with addition, subtraction, and multiplication of integers.**

    Objective 7.1.1: Identify numbers less than zero by extending the number line.
    Objective 7.1.2: Solve problems involving basic operations on whole numbers, including addition and subtraction of seven-digit numbers, multiplication with two-digit multipliers, and division with two-digit divisors.

    Additional content to be taught:
    - Developing algorithms for performing operations on integers
        - Example: determining a systematic set of steps that can be used to subtract integers
    - Using inverse properties of addition and of multiplication

2. **Use order of operations to evaluate numerical expressions.**

    Objective 7.2.1: Demonstrate the commutative, associative, and identity properties of addition and multiplication of whole numbers.
    Objective 7.2.2: Apply the distributive property to evaluate numerical expressions.

    Additional content to be taught:
    - Computing absolute values
    - Finding square roots of perfect squares through 225
    - Evaluating powers
        - Example: $2^3 = 8$
    - Applying properties of operations to compute with integers, fractions, and decimals

3. **Solve problems requiring the use of operations on rational numbers.**

   Objective 7.3.1: Demonstrate computational fluency with addition, subtraction, multiplication, and division of decimals and fractions.
   Objective 7.3.2: Solve problems involving decimals, percents, fractions, and proportions.

   Additional content to be taught:
   - Describing the method used
   - Determining the reasonableness of results
   - Using percents to solve problems, including problems involving discounts, taxes, commissions, and simple interest

Algebra

4. **Express a pattern shown in a table, graph, or chart as an algebraic equation.**

   Objective 7.4.1: Solve problems using numeric and geometric patterns.
   Objective 7.4.2: Determine a verbal rule for a function given the input and output.

   Additional content to be taught:
   - Recognizing the relationships between numerical patterns in tables and their respective graphs in the coordinate plane
       - Example: recognizing the pattern illustrated in both the table and graph as $y = 2x$

x	y
1	2
2	4
3	6
4	8
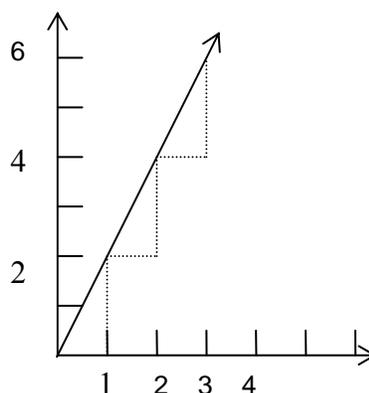
   - Determining if a constant rate of change exists in a pattern

5. **Translate verbal phrases into algebraic expressions and algebraic expressions into verbal phrases.**

   Objective 7.5.1: Write a number sentence for a problem expressed in words.
   Objective 7.5.2: Write a problem expressed in words for a number sentence.

   Additional content to be taught:
   - Exhibiting understanding of a variable as an unknown quantity

6. **Solve one- and two-step equations.**

   Objective 7.6.1:   Solve problems involving addition, subtraction, multiplication, and division of decimals and fractions.
   Objective 7.6.2:   Demonstrate the commutative, associative, and identity properties of addition and multiplication of whole numbers.

   Additional content to be taught:
   - Solving inequalities in one variable
   - Graphing solution sets of inequalities on a number line
   - Recognizing properties of equality

Geometry

7. **Determine the transformation(s), including translations, reflections, or rotations, used to alter the position of a polygon on the coordinate plane.**

   Objective 7.7.1:   Plot coordinates on grids, graphs, and maps.
   Objective 7.7.2:   Identify figures that have rotational symmetry.
   Objective 7.7.3:   Predict the results of a flip (reflection), turn (rotation), or slide (translation).

   Additional content to be taught:
   - Determining the type of symmetry (rotational or line) found in a reflection or rotation
   - Graphing transformations of quadrilaterals on the Cartesian plane by plotting the vertices
   - Graphing figures which are similar to other figures using dilations

8. **Recognize geometric relationships among two-dimensional and three-dimensional objects.**

   Objective 7.8.1:   Identify two-dimensional and three-dimensional figures based on attributes, properties, and component parts.
   Objective 7.8.2:   Identify regular polygons and congruent polygons.
   Objective 7.8.3:   Measure length and width, using metric and customary units.

   Additional content to be taught:
   - Drawing geometric figures when given specified components, including base and height
   - Investigating properties and relationships among congruent figures
   - Identifying geometric ideas in settings outside the mathematics classroom
   - Using networks to represent and solve problems

Measurement

9. **Solve problems involving circumference and area of circles.**

   Objective 7.9.1: Identify the center, radius, and diameter of a circle.
   Objective 7.9.2: Measure length and width, using metric and customary units.

   Additional content to be taught:
   - Estimating circumference, diameter, and area
   - Determining appropriate units of measure to describe circumference, diameter, and area
   - Measuring circumference and diameter using customary and metric units
   - Using circumference and diameter to approximate the value of π
   - Identifying π as an irrational number
   - Developing formulas for determining circumference and area

10. **Find the perimeter of polygons and the area of triangles and trapezoids.**

    Objective 7.10.1: Develop formulas to determine perimeter and area of parallelograms and rectangles.
    Objective 7.10.2: Estimate perimeter and area of irregular shapes using unit squares and grid paper.
    Objective 7.10.3: Calculate the perimeter of rectangles from measured dimensions.

    Additional content to be taught:
    - Developing formulas for determining perimeter and area of triangles and trapezoids

11. **Solve problems involving ratios or rates, using proportional reasoning.**

    Objective 7.11.1: Convert units of length, weight, or capacity within the same system (customary or metric).
    Objective 7.11.2: Solve problems involving decimals, percents, fractions, and proportions.

    Additional content to be taught:
    - Determining the unit rate
    - Converting rates from one unit to another
          Example: determining the number of minutes in three days
    - Converting units of length, weight, or capacity from metric to customary and from customary to metric

Data Analysis and Probability

12. **Determine measures of central tendency (mean, median, and mode) and the range using a given set of data or graphs, including histograms, frequency tables, and stem-and-leaf plots.**

    Objective 7.12.1: Interpret information from bar graphs, line graphs, and circle graphs.
    Objective 7.12.2: Demonstrate computational fluency with addition, subtraction, multiplication, and division of decimals and fractions.
    Objective 7.12.3: Analyze data collected from a survey or experiment to distinguish between what the data show and what might account for the results.

    Additional content to be taught:
    - Creating histograms

13. **Determine the probability of a compound event.**
    Example: finding the probability of selecting at random a hamburger and lemonade when choosing a sandwich and drink from two sandwich choices and three drink choices

    Objective 7.13.1: Find the probability of a simple event.
    Objective 7.13.2: Use common fractions to represent the probability of events that are neither certain nor impossible.

    Additional content to be taught:
    - Representing outcomes as a list, chart, picture, or tree diagram
    - Determining the number of possible outcomes by using the fundamental counting principle or other techniques
    - Modeling the probability of events through simulations with random numbers
        Example: determining the probability of a baby being a boy by generating random numbers, using a number cube with odd numbers representing a boy, to simulate the outcomes

Eighth Grade
(Pre-Algebra)

Number and Operations

Students will:

1. **Use various strategies and operations to solve problems involving real numbers.**

    Objective 8.1.1: Demonstrate computational fluency with addition, subtraction, multiplication, and division of integers.
    Objective 8.1.2: Solve problems involving ratios or rates, using proportional reasoning.

    Additional content to be taught:
    - Using alternative representations of rational numbers
    - Applying GCF, LCM, and prime and composite numbers, including justification for the reasonableness of results, when working with rational numbers
    - Applying proportional reasoning
    - Using vocabulary associated with sets, including *union* and *intersection*
    - Determining whether a number is rational or irrational
    - Demonstrating computational fluency with operations on rational numbers

2. **Simplify expressions containing natural number exponents by applying one or more of the laws of exponents.**

    Objective 8.2.1: Evaluate powers.

    Additional content to be taught:
    - Writing numbers using scientific notation

3. **Use order of operations to evaluate and simplify algebraic expressions.**

    Objective 8.3.1: Develop algorithms for performing operations on integers.
             Example: determining a systematic set of steps that can be used to subtract integers
    Objective 8.3.2: Apply properties of operations to compute with integers, fractions, and decimals.

    Additional content to be taught:
    - Applying the substitution principle
    - Applying the properties of operations on rational numbers to evaluate and simplify algebraic expressions

Algebra

4. **Graph linear relations by plotting points or by using the slope and y-intercept.**

   Objective 8.4.1: Recognize the relationships between numerical patterns in tables and their respective graphs in the coordinate plane.
   Objective 8.4.2: Plot coordinates on grids, graphs, and maps.
   Objective 8.4.3: Identify the coordinates of a point on the Cartesian plane.

   Additional content to be taught:
   - Determining slopes and y-intercepts of lines
   - Calculating the slope of a linear relation given as a table or graph
   - Exhibiting conceptual understanding of various uses of variables

5. **Solve problems involving linear functions.**

   Objective 8.5.1: Express a pattern shown in a table, graph, or chart as an algebraic equation.
   Objective 8.5.2: Determine verbal rule for a function given the input and output.

   Additional content to be taught:
   - Identifying functions from information in tables, sets of ordered pairs, equations, graphs, and mappings
   - Determining the rule that defines a function

     Example:

Cars Washed Input (x)	Money Made Output (y)
1	$4.50
2	$9.00
3	$13.50
4	$18.00

     (Rule: $y = 4.5x$)
   - Classifying variables in a function as independent or dependent
   - Classifying relations as linear or nonlinear by examining tables, graphs, or simple equations

6. **Solve multistep linear equations, including equations requiring the use of the distributive property.**

   Example: solving $-3(x - 5) - 6x = 2 + 4x$

   Objective 8.6.1:   Solve one- and two-step equations.
   Objective 8.6.2:   Recognize properties of equality.
   Objective 8.6.3:   Solve problems requiring the use of operations on rational numbers.
   Objective 8.6.4:   Apply the distributive property to compute with fractions and decimals.

Geometry

7. **Solve problems using the Pythagorean Theorem.**

   Objective 8.7.1:   Evaluate powers.
   Objective 8.7.2:   Use order of operations to evaluate numerical expressions.
   Objective 8.7.3:   Solve problems requiring the use of operations on rational numbers.

   Additional content to be taught:
   - Applying the Triangle Inequality Theorem
        Example: determining if a triangle can be formed with sides of 1 inch, 2 inches, and 5 inches
   - Verifying the Pythagorean Theorem
   - Applying the Pythagorean Theorem to determine if a triangle is a right triangle
   - Applying the Pythagorean Theorem to find the missing length of a side of a right triangle
   - Calculating distances on the coordinate plane using the Pythagorean Theorem

8. **Compare quadrilaterals, triangles, and solids, using their properties and characteristics.**

   Objective 8.8.1: Identify properties and relationships among congruent figures.
   Objective 8.8.2: Classify angles as acute, obtuse, right, or straight.
   Objective 8.8.3: Measure angles.

   Additional content to be taught:
   - Developing mathematical arguments about the relationships among types of quadrilaterals and triangles
   - Identifying angle bisectors, perpendiculars bisectors, congruent angles, and congruent figures
   - Constructing congruent and similar polygons, congruent angles, congruent segments, and parallel and perpendicular lines

Measurement

9. **Determine the measures of special angle pairs, including adjacent, vertical, supplementary, and complementary angles, and angles formed by parallel lines cut by a transversal.**
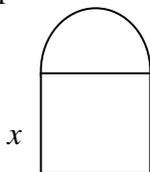
   Objective 8.9.1: Compare parallel and perpendicular lines.
   Objective 8.9.2: Classify angles as acute, obtuse, right, or straight.
   Objective 8.9.3: Measure angles.

10. **Find the perimeter and area of regular and irregular plane figures.**
    Example:

    If x represents the length of a side of the square, write expressions that represent the perimeter and area of the figure at the left.

    Objective 8.10.1: Find the perimeter of polygons and the area of triangles and trapezoids.
    Objective 8.10.2: Develop formulas to determine perimeter and area of parallelograms and rectangles.

11. **Determine the surface area and volume of rectangular prisms, cylinders, and pyramids.**

Objective 8.11.1: Develop formulas to determine perimeter and area of triangles and trapezoids.

Objective 8.11.2: Recognize geometric relationships among two-dimensional and three-dimensional objects.

Objective 8.11.3: Develop formulas to determine perimeter and area of parallelograms and rectangles.

Additional content to be taught:
- Estimating surface area and volume of solid figures
- Determining the appropriate units of measure to describe surface area and volume
- Developing formulas for determining surface area and volume of rectangular prisms, cylinders, and pyramids

12. **Determine the lengths of missing sides and measures of angles in similar and congruent figures.**

Objective 8.12.1: Solve problems involving ratios or rates, using proportional reasoning.

Objective 8.12.2: Determine the transformation(s), including translations, reflections, or rotations used to alter the position of a polygon on the coordinate plane.

Additional content to be taught:
- Applying proportional reasoning
- Using dilations on the coordinate plane to determine measures of similar figures
- Finding the ratios of the perimeters and areas of similar triangles, trapezoids, and parallelograms

Data Analysis and Probability

13. **Interpret data from populations, using given and collected data.**

Objective 8.13.1: Determine measures of central tendency (mean, median, and mode) and the range using a given set of data or graphs, including histograms, frequency tables, and stem-and-leaf plots.

Objective 8.13.2: Solve problems involving decimals, percents, fractions, and proportions.

Additional content to be taught:
- Representing the data with the most appropriate graph, including box-and-whisker plot, circle graph, and scatterplot
- Making predictions by estimating the line of best fit from a scatterplot
- Comparing data sets involving two populations
- Determining the measure of center that is the most appropriate for a given situation

14. **Determine the theoretical probability of an event.**

Objective 8.14.1: Find the probability of a simple event.
Objective 8.14.2: Determine the probability of a compound event.
Objective 8.14.3: Express probabilities as ratios, percents, and decimals.
Objective 8.14.4: Represent outcomes as a list, chart, picture, or tree diagram.

Additional content to be taught:
- Calculating the probability of complementary events and mutually exclusive events
- Comparing experimental and theoretical probability
- Computing the probability of two independent events and two dependent events
- Determining the probability of an event through simulation
    - Example: using random numbers to find the probability of a basketball player making six baskets in six attempts if he makes 60 percent of his shots from the court and shoots 20 times during a game

Appendix A

Table of Mathematics Standards and Expectations

Prekindergarten – Grade 12*

<div style="background:black;color:white;text-align:center;font-weight:bold">NUMBER AND OPERATIONS STANDARD</div>

Instructional programs from prekindergarten through grade 12 should enable all students to—

Understand numbers, ways of representing numbers, relationships among numbers, and number systems

Pre-K-2 Expectations:
In prekindergarten through grade 2 all students should—

- count with understanding and recognize "how many" in sets of objects;
- use multiple models to develop initial understandings of place value and the base-ten number system;
- develop understanding of the relative position and magnitude of whole numbers and of ordinal and cardinal numbers and their connections;
- develop a sense of whole numbers and represent and use them in flexible ways, including relating, composing, and decomposing numbers;
- connect number words and numerals to the quantities they represent, using various physical models and representations;
- understand and represent commonly used fractions, such as 1/4, 1/3, and 1/2.

Grades 3-5 Expectations:
In grades 3-5 all students should—

- understand the place-value structure of the base-ten number system and be able to represent and compare whole numbers and decimals;
- recognize equivalent representations for the same number and generate them by decomposing and composing numbers;
- develop understanding of fractions as parts of unit wholes, as parts of a collection, as locations on number lines, and as divisions of whole numbers;
- use models, benchmarks, and equivalent forms to judge the size of fractions;
- recognize and generate equivalent forms of commonly used fractions, decimals, and percents;
- explore numbers less than 0 by extending the number line and through familiar applications;
- describe classes of numbers according to characteristics such as the nature of their factors.

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- work flexibly with fractions, decimals, and percents to solve problems;
- compare and order fractions, decimals, and percents efficiently and find their approximate locations on a number line;
- develop meaning for percents greater than 100 and less than 1;
- understand and use ratios and proportions to represent quantitative relationships;
- develop an understanding of large numbers and recognize and appropriately use exponential, scientific, and calculator notation;
- use factors, multiples, prime factorization, and relatively prime numbers to solve problems;
- develop meaning for integers and represent and compare quantities with them.

- develop a deeper understanding of very large and very small numbers and of various representations of them;
- compare and contrast the properties of numbers and number systems, including the rational and real numbers, and understand complex numbers as solutions to quadratic equations that do not have real solutions;
- understand vectors and matrices as systems that have some of the properties of the real-number system;
- use number-theory arguments to justify relationships involving whole numbers.

Understand meanings of operations and how they relate to one another

- understand various meanings of addition and subtraction of whole numbers and the relationship between the two operations;
- understand the effects of adding and subtracting whole numbers;
- understand situations that entail multiplication and division, such as equal groupings of objects and sharing equally.

- understand various meanings of multiplication and division;
- understand the effects of multiplying and dividing whole numbers;
- identify and use relationships between operations, such as division as the inverse of multiplication, to solve problems;
- understand and use properties of operations, such as the distributivity of multiplication over addition.

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- understand the meaning and effects of arithmetic operations with fractions, decimals, and integers;
- use the associative and commutative properties of addition and multiplication and the distributive property of multiplication over addition to simplify computations with integers, fractions, and decimals;
- understand and use the inverse relationships of addition and subtraction, multiplication and division, and squaring and finding square roots to simplify computations and solve problems.

- judge the effects of such operations as multiplication, division, and computing powers and roots on the magnitudes of quantities;
- develop an understanding of properties of, and representations for, the addition and multiplication of vectors and matrices;
- develop an understanding of permutations and combinations as counting techniques.

Compute fluently and make reasonable estimates

- develop and use strategies for whole-number computations, with a focus on addition and subtraction;
- develop fluency with basic number combinations for addition and subtraction;
- use a variety of methods and tools to compute, including objects, mental computation, estimation, paper and pencil, and calculators.

- develop fluency with basic number combinations for multiplication and division and use these combinations to mentally compute related problems, such as 30 x 50;
- develop fluency in adding, subtracting, multiplying, and dividing whole numbers;
- develop and use strategies to estimate the results of whole-number computations and to judge the reasonableness of such results;
- develop and use strategies to estimate computations involving fractions and decimals in situations relevant to students' experience;
- use visual models, benchmarks, and equivalent forms to add and subtract commonly used fractions and decimals;
- select appropriate methods and tools for computing with whole numbers from among mental computation, estimation, calculators, and paper and pencil according to the context and nature of the computation and use the selected method or tools.

Grades 6-8 Expectations:
In grades 6-8 all students should—

- select appropriate methods and tools for computing with fractions and decimals from among mental computation, estimation, calculators or computers, and paper and pencil, depending on the situation, and apply the selected methods;
- develop and analyze algorithms for computing with fractions, decimals, and integers and develop fluency in their use;
- develop and use strategies to estimate the results of rational-number computations and judge the reasonableness of the results;
- develop, analyze, and explain methods for solving problems involving proportions, such as scaling and finding equivalent ratios.

Grades 9-12 Expectations:
In grades 9-12 all students should—

- develop fluency in operations with real numbers, vectors, and matrices, using mental computation or paper-and-pencil calculations for simple cases and technology for more-complicated cases.
- judge the reasonableness of numerical computations and their results.

ALGEBRA STANDARD

Instructional programs from prekindergarten through grade 12 should enable all students to—

Understand patterns, relations, and functions

Pre-K-2 Expectations:
In prekindergarten through grade 2 all students should—

- sort, classify, and order objects by size, number, and other properties;
- recognize, describe, and extend patterns such as sequences of sounds and shapes or simple numeric patterns and translate from one representation to another;
- analyze how both repeating and growing patterns are generated.

Grades 3-5 Expectations:
In grades 3-5 all students should—

- describe, extend, and make generalizations about geometric and numeric patterns;
- represent and analyze patterns and functions, using words, tables, and graphs.

Grades 6-8 Expectations:
In grades 6-8 all students should—

- represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic rules;
- relate and compare different forms of representation for a relationship;
- identify functions as linear or nonlinear and contrast their properties from tables, graphs, or equations.

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- generalize patterns using explicitly defined and recursively defined functions;
- understand relations and functions and select, convert flexibly among, and use various representations for them;
- analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior;
- understand and perform transformations such as arithmetically combining, composing, and inverting commonly used functions, using technology to perform such operations on more-complicated symbolic expressions;
- understand and compare the properties of classes of functions, including exponential, polynomial, rational, logarithmic, and periodic functions;
- interpret representations of functions of two variables

Represent and analyze mathematical situations and structures using algebraic symbols

Pre-K-2 Expectations:
In prekindergarten through grade 2 all students should—

- illustrate general principles and properties of operations, such as commutativity, using specific numbers;
- use concrete, pictorial, and verbal representations to develop an understanding of invented and conventional symbolic notations.

Grades 3-5 Expectations:
In grades 3-5 all students should—

- identify such properties as commutativity, associativity, and distributivity and use them to compute with whole numbers;
- represent the idea of a variable as an unknown quantity using a letter or a symbol;
- express mathematical relationships using equations.

Grades 6-8 Expectations:
In grades 6-8 all students should—

- develop an initial conceptual understanding of different uses of variables;
- explore relationships between symbolic expressions and graphs of lines, paying particular attention to the meaning of intercept and slope;
- use symbolic algebra to represent situations and to solve problems, especially those that involve linear relationships;
- recognize and generate equivalent forms for simple algebraic expressions and solve linear equations

Grades 9-12 Expectations:
In grades 9-12 all students should—

- understand the meaning of equivalent forms of expressions, equations, inequalities, and relations;
- write equivalent forms of equations, inequalities, and systems of equations and solve them with fluency–mentally or with paper and pencil in simple cases and using technology in all cases;
- use symbolic algebra to represent and explain mathematical relationships;
- use a variety of symbolic representations, including recursive and parametric equations, for functions and relations;
- judge the meaning, utility, and reasonableness of the results of symbol manipulations, including those carried out by technology.

Use mathematical models to represent and understand quantitative relationships

Pre-K-2 Expectations:
In prekindergarten through grade 2 all students should—

- model situations that involve the addition and subtraction of whole numbers, using objects, pictures, and symbols.

Grades 3-5 Expectations:
In grades 3-5 all students should—

- model problem situations with objects and use representations such as graphs, tables, and equations to draw conclusions.

Grades 6-8 Expectations:
In grades 6-8 all students should—

- model and solve contextualized problems using various representations, such as graphs, tables, and equations.

Grades 9-12 Expectations:
In grades 9-12 all students should—

- identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationships;
- use symbolic expressions, including iterative and recursive forms, to represent relationships arising from various contexts;
- draw reasonable conclusions about a situation being modeled.

Analyze change in various contexts

Pre-K-2 Expectations:
In prekindergarten through grade 2 all students should—

- describe qualitative change, such as a student's growing taller;
- describe quantitative change, such as a student's growing two inches in one year.

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Grades 3-5 Expectations:
In grades 3-5 all students should—

- investigate how a change in one variable relates to a change in a second variable;
- identify and describe situations with constant or varying rates of change and compare them.

Grades 6-8 Expectations:
In grades 6-8 all students should—

- use graphs to analyze the nature of changes in quantities in linear relationships.

Grades 9-12 Expectations:
In grades 9-12 all students should—

- approximate and interpret rates of change from graphical and numerical data.

GEOMETRY STANDARD

Instructional programs from prekindergarten through grade 12 should enable all students to—

Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships

Pre-K-2 Expectations:
In prekindergarten through grade 2 all students should—

- recognize, name, build, draw, compare, and sort two- and three-dimensional shapes;
- describe attributes and parts of two- and three-dimensional shapes;
- investigate and predict the results of putting together and taking apart two- and three-dimensional shapes.

Grades 3-5 Expectations:
In grades 3-5 all students should—

- identify, compare, and analyze attributes of two- and three-dimensional shapes and develop vocabulary to describe the attributes;
- classify two- and three-dimensional shapes according to their properties and develop definitions of classes of shapes such as triangles and pyramids;
- investigate, describe, and reason about the results of subdividing, combining, and transforming shapes;
- explore congruence and similarity;
- make and test conjectures about geometric properties and relationships and develop logical arguments to justify conclusions.

Grades 6-8 Expectations:
In grades 6-8 all students should—

- precisely describe, classify, and understand relationships among types of two- and three-dimensional objects using their defining properties;
- understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects;
- create and critique inductive and deductive arguments concerning geometric ideas and relationships, such as congruence, similarity, and the Pythagorean relationship.

Grades 9-12 Expectations:
In grades 9-12 all students should—

- analyze properties and determine attributes of two- and three-dimensional objects;
- explore relationships (including congruence and similarity) among classes of two- and three-dimensional geometric objects, make and test conjectures about them, and solve problems involving them;
- establish the validity of geometric conjectures using deduction, prove theorems, and critique arguments made by others;
- use trigonometric relationships to determine lengths and angle measures.

Specify locations and describe spatial relationships using coordinate geometry and other representational systems

Pre-K-2 Expectations:
In prekindergarten through grade 2 all students should—

- describe, name, and interpret relative positions in space and apply ideas about relative position;
- describe, name, and interpret direction and distance in navigating space and apply ideas about direction and distance;
- find and name locations with simple relationships such as "near to" and in coordinate systems such as maps.

Grades 3-5 Expectations:
In grades 3-5 all students should—

- describe location and movement using common language and geometric vocabulary;
- make and use coordinate systems to specify locations and to describe paths;
- find the distance between points along horizontal and vertical lines of a coordinate system.

Grades 6-8 Expectations:
In grades 6-8 all students should—

- use coordinate geometry to represent and examine the properties of geometric shapes;
- use coordinate geometry to examine special geometric shapes, such as regular polygons or those with pairs of parallel or perpendicular sides.

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- use Cartesian coordinates and other coordinate systems, such as navigational, polar, or spherical systems, to analyze geometric situations;
- investigate conjectures and solve problems involving two- and three-dimensional objects represented with Cartesian coordinates.

Apply transformations and use symmetry to analyze mathematical situations

Pre-K-2 Expectations:
In prekindergarten through grade 2 all students should—

- recognize and apply slides, flips, and turns;
- recognize and create shapes that have symmetry.

Grades 3-5 Expectations:
In grades 3-5 all students should—

- predict and describe the results of sliding, flipping, and turning two-dimensional shapes;
- describe a motion or a series of motions that will show that two shapes are congruent;
- identify and describe line and rotational symmetry in two- and three-dimensional shapes and designs.

Grades 6-8 Expectations:
In grades 6-8 all students should—

- describe sizes, positions, and orientations of shapes under informal transformations such as flips, turns, slides, and scaling;
- examine the congruence, similarity, and line or rotational symmetry of objects using transformations.

Grades 9-12 Expectations:
In grades 9-12 all students should—

- understand and represent translations, reflections, rotations, and dilations of objects in the plane by using sketches, coordinates, vectors, function notation, and matrices;
- use various representations to help understand the effects of simple transformations and their compositions.

Use visualization, spatial reasoning, and geometric modeling to solve problems

Pre-K-2 Expectations:
In prekindergarten through grade 2 all students should—

- create mental images of geometric shapes using spatial memory and spatial visualization;
- recognize and represent shapes from different perspectives;
- relate ideas in geometry to ideas in number and measurement;
- recognize geometric shapes and structures in the environment and specify their location.

Grades 3-5 Expectations:
In grades 3-5 all students should—

- build and draw geometric objects;
- create and describe mental images of objects, patterns, and paths;
- identify and build a three-dimensional object from two-dimensional representations of that object;
- identify and draw a two-dimensional representation of a three-dimensional object;
- use geometric models to solve problems in other areas of mathematics, such as number and measurement;
- recognize geometric ideas and relationships and apply them to other disciplines and to problems that arise in the classroom or in everyday life.

Grades 6-8 Expectations:
In grades 6-8 all students should—

- draw geometric objects with specified properties, such as side lengths or angle measures;
- use two-dimensional representations of three-dimensional objects to visualize and solve problems such as those involving surface area and volume;
- use visual tools such as networks to represent and solve problems;
- use geometric models to represent and explain numerical and algebraic relationships;
- recognize and apply geometric ideas and relationships in areas outside the mathematics classroom, such as art, science, and everyday life.

Grades 9-12 Expectations:
In grades 9-12 all students should—

- draw and construct representations of two- and three-dimensional geometric objects using a variety of tools;
- visualize three-dimensional objects and spaces from different perspectives and analyze their cross sections;
- use vertex-edge graphs to model and solve problems;
- use geometric models to gain insights into, and answer questions in, other areas of mathematics;
- use geometric ideas to solve problems in, and gain insights into, other disciplines and other areas of interest such as art and architecture.

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MEASUREMENT STANDARD

Instructional programs from prekindergarten through grade 12 should enable all students to—

Understand measurable attributes of objects and the units, systems, and processes of measurement

Pre-K-2 Expectations:
In prekindergarten through grade 2 all students should—

- recognize the attributes of length, volume, weight, area, and time;
- compare and order objects according to these attributes;
- understand how to measure using nonstandard and standard units;
- select an appropriate unit and tool for the attribute being measured.

Grades 3-5 Expectations:
In grades 3-5 all students should—

- understand such attributes as length, area, weight, volume, and size of angle and select the appropriate type of unit for measuring each attribute;
- understand the need for measuring with standard units and become familiar with standard units in the customary and metric systems;
- carry out simple unit conversions, such as from centimeters to meters, within a system of measurement;
- understand that measurements are approximations and how differences in units affect precision;
- explore what happens to measurements of a two-dimensional shape such as its perimeter and area when the shape is changed in some way.

Grades 6-8 Expectations:
In grades 6-8 all students should—

- understand both metric and customary systems of measurement;
- understand relationships among units and convert from one unit to another within the same system;
- understand, select, and use units of appropriate size and type to measure angles, perimeter, area, surface area, and volume.

Grades 9-12 Expectations:
In grades 9-12 all students should—

- make decisions about units and scales that are appropriate for problem situations involving measurement.

Apply appropriate techniques, tools, and formulas to determine measurements.

Pre-K-2 Expectations:
In prekindergarten through grade 2 all students should—

- measure with multiple copies of units of the same size, such as paper clips laid end to end;
- use repetition of a single unit to measure something larger than the unit, for instance, measuring the length of a room with a single meter stick;
- use tools to measure;
- develop common referents for measures to make comparisons and estimates.

Grades 3-5 Expectations:
In grades 3-5 all students should—

- develop strategies for estimating the perimeters, areas, and volumes of irregular shapes;
- select and apply appropriate standard units and tools to measure length, area, volume, weight, time, temperature, and the size of angles;
- select and use benchmarks to estimate measurements;
- develop, understand, and use formulas to find the area of rectangles and related triangles and parallelograms;
- develop strategies to determine the surface areas and volumes of rectangular solids.

Grades 6-8 Expectations:
In grades 6-8 all students should—

- use common benchmarks to select appropriate methods for estimating measurements;
- select and apply techniques and tools to accurately find length, area, volume, and angle measures to appropriate levels of precision;
- develop and use formulas to determine the circumference of circles and the area of triangles, parallelograms, trapezoids, and circles and develop strategies to find the area of more complex shapes;
- develop strategies to determine the surface area and volume of selected prisms, pyramids, and cylinders;
- solve problems involving scale factors, using ratio and proportion;
- solve simple problems involving rates and derived measurements for such attributes as velocity and density.

Grades 9-12 Expectations:
In grades 9-12 all students should—

- analyze precision, accuracy, and approximate error in measurement situations;
- understand and use formulas for the area, surface area, and volume of geometric figures, including cones, spheres, and cylinders;
- apply informal concepts of successive approximation, upper and lower bounds, and limit in measurement situations;
- use unit analysis to check measurement computations.

Appendix A

DATA ANALYSIS AND PROBABILITY STANDARD

Instructional programs from prekindergarten through grade 12 should enable all students to—

Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them

Pre-K-2 Expectations:
In prekindergarten through grade 2 all students should—

- pose questions and gather data about themselves and their surroundings;
- sort and classify objects according to their attributes and organize data about the objects;
- represent data using concrete objects, pictures, and graphs.

Grades 3-5 Expectations:
In grades 3-5 all students should—

- design investigations to address a question and consider how data-collection methods affect the nature of the data set;
- collect data using observations, surveys, and experiments;
- represent data using tables and graphs such as line plots, bar graphs, and line graphs;
- recognize the differences in representing categorical and numerical data.

Grades 6-8 Expectations:
In grades 6-8 all students should—

- formulate questions, design studies, and collect data about a characteristic shared by two populations or different characteristics within one population;
- select, create, and use appropriate graphical representations of data, including histograms, box plots, and scatterplots.

Grades 9-12 Expectations:
In grades 9-12 all students should—

- understand the differences among various kinds of studies and which types of inferences can legitimately be drawn from each;
- know the characteristics of well-designed studies, including the role of randomization in surveys and experiments;
- understand the meaning of measurement data and categorical data, of univariate and bivariate data, and of the term variable;
- understand histograms, parallel box plots, and scatterplots and use them to display data;
- compute basic statistics and understand the distinction between a statistic and a parameter.

Select and use appropriate statistical methods to analyze data

Pre-K-2 Expectations:
In prekindergarten through grade 2 all students should—

- describe parts of the data and the set of data as a whole to determine what the data show.

Grades 3-5 Expectations:
In grades 3-5 all students should—

- describe the shape and important features of a set of data and compare related data sets, with an emphasis on how the data are distributed;
- use measures of center, focusing on the median, and understand what each does and does not indicate about the data set;
- compare different representations of the same data and evaluate how well each representation shows important aspects of the data.

Grades 6-8 Expectations:
In grades 6-8 all students should—

- find, use, and interpret measures of center and spread, including mean and interquartile range;
- discuss and understand the correspondence between data sets and their graphical representations, especially histograms, stem-and-leaf plots, box plots, and scatterplots.

Grades 9-12 Expectations:
In grades 9-12 all students should—

- for univariate measurement data, be able to display the distribution, describe its shape, and select and calculate summary statistics;
- for bivariate measurement data, be able to display a scatterplot, describe its shape, and determine regression coefficients, regression equations, and correlation coefficients using technological tools;
- display and discuss bivariate data where at least one variable is categorical;
- recognize how linear transformations of univariate data affect shape, center, and spread;
- identify trends in bivariate data and find functions that model the data or transform the data so that they can be modeled.

Develop and evaluate inferences and predictions that are based on data

Pre-K-2 Expectations:
In prekindergarten through grade 2 all students should—

- discuss events related to students' experiences as likely or unlikely.

Grades 3-5 Expectations:
In grades 3-5 all students should—

- propose and justify conclusions and predictions that are based on data and design studies to further investigate the conclusions or predictions.

Grades 6-8 Expectations:
In grades 6-8 all students should—

- use observations about differences between two or more samples to make conjectures about the populations from which the samples were taken;
- make conjectures about possible relationships between two characteristics of a sample on the basis of scatterplots of the data and approximate lines of fit;
- use conjectures to formulate new questions and plan new studies to answer them.

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- use simulations to explore the variability of sample statistics from a known population and to construct sampling distributions;
- understand how sample statistics reflect the values of population parameters and use sampling distributions as the basis for informal inference;
- evaluate published reports that are based on data by examining the design of the study, the appropriateness of the data analysis, and the validity of conclusions;
- understand how basic statistical techniques are used to monitor process characteristics in the workplace.

Understand and apply basic concepts of probability

Pre-K-2 Expectations:
In prekindergarten through grade 2 all students should—

Grades 3-5 Expectations:
In grades 3-5 all students should—

- describe events as likely or unlikely and discuss the degree of likelihood using such words as *certain*, *equally likely*, and *impossible*;
- predict the probability of outcomes of simple experiments and test the predictions;
- understand that the measure of the likelihood of an event can be represented by a number from 0 to 1.

Grades 6-8 Expectations:
In grades 6-8 all students should—

- understand and use appropriate terminology to describe complementary and mutually exclusive events;
- use proportionality and a basic understanding of probability to make and test conjectures about the results of experiments and simulations;
- compute probabilities for simple compound events, using such methods as organized lists, tree diagrams, and area models.

Grades 9-12 Expectations:
In grades 9-12 all students should—

- understand the concepts of sample space and probability distribution and construct sample spaces and distributions in simple cases;
- use simulations to construct empirical probability distributions;
- compute and interpret the expected value of random variables in simple cases;
- understand the concepts of conditional probability and independent events;
- understand how to compute the probability of a compound event.

PROBLEM-SOLVING PROCESS

Instructional programs from prekindergarten through grade 12 should enable all students to—

- Build new mathematical knowledge through problem solving
- Solve problems that arise in mathematics and in other contexts
- Apply and adapt a variety of appropriate strategies to solve problems
- Monitor and reflect on the process of mathematical problem solving

REASONING AND PROOF PROCESS

Instructional programs from prekindergarten through grade 12 should enable all students to—

- Recognize reasoning and proof as fundamental aspects of mathematics
- Make and investigate mathematical conjectures
- Develop and evaluate mathematical arguments and proofs
- Select and use various types of reasoning and methods of proof

COMMUNICATION PROCESS

Instructional programs from prekindergarten through grade 12 should enable all students to—

- Organize and consolidate their mathematical thinking through communication
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others
- Analyze and evaluate the mathematical thinking and strategies of others
- Use the language of mathematics to express mathematical ideas precisely

CONNECTIONS PROCESS

Instructional programs from prekindergarten through grade 12 should enable all students to—

- Recognize and use connections among mathematical ideas
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole
- Recognize and apply mathematics in contexts outside of mathematics

REPRESENTATION PROCESS

Instructional programs from prekindergarten through grade 12 should enable all students to—

- Create and use representations to organize, record, and communicate mathematical ideas
- Select, apply, and translate among mathematical representations to solve problems
- Use representations to model and interpret physical, social, and mathematical phenomena

Appendix B

Standards and Objectives
Alabama High School Graduation Exam
Mathematics

STANDARD I. The student will be able to perform basic operations on algebraic expressions.

OBJECTIVE

1. Apply order of operations.

2. Add and subtract polynomials.

3. Multiply polynomials.

4. Factor polynomials.

STANDARD II. The student will be able to solve equations and inequalities.

OBJECTIVE

1. Solve multi-step equations of first degree.

2. Solve quadratic equations that are factorable.

3. Solve systems of two linear equations.

4. Solve multi-step inequalities of first degree.

STANDARD III. The student will be able to apply concepts related to functions.

OBJECTIVE

1. Identify functions.

2. Find the range of functions when given the domain.

STANDARD IV. The student will be able to apply formulas.

OBJECTIVE

1. Find the perimeter, circumference, area, or volume of geometric figures.

2. Find the distance, midpoint, or slope of line segments when given two points.

STANDARD V. The student will be able to apply graphing techniques.

OBJECTIVE

1. Graph or identify graphs of linear equations.

2. Graph lines given certain conditions.

3. Determine solution sets of inequalities.

4. Identify graphs of common relations.

STANDARD VI. The student will be able to represent problem situations.

OBJECTIVE

1. Translate verbal or symbolic information into algebraic expressions; or identify equations or inequalities that represent graphs or problem situations.

STANDARD VII. The student will be able to solve problems involving a variety of algebraic and geometric concepts.

OBJECTIVE

1. Apply properties of angles and relationships between angles.

2. Apply Pythagorean Theorem.

3. Apply properties of similar polygons.

4. Apply properties of plane and solid geometric figures.

5. Determine measures of central tendency.

6. Determine probabilities.

7. Solve problems involving direct variation.

8. Solve problems involving algebraic concepts.

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Glossary

Algorithm. A set of steps used to carry out a computation.

Angle bisector. A ray that divides an angle into two congruent angles.

Associative property of addition. If a, b, and c are real numbers, then $a + (b + c) = (a + b) + c$. (*The same terms in the same order added in different groupings yield the same results*.)

Associative property of multiplication. If a, b, and c are real numbers, then $a(bc) = (ab)c$. (*The same factors in the same order multiplied in different groupings yield the same results*.)

Cartesian plane. The coordinate plane formed by two perpendicular number lines intersecting at the origin, (0,0).

Common factor. A factor shared by all terms in an algebraic expression.

Commutative property of addition. If a and b are real numbers, then $a + b = b + a$. (*The order in which two numbers are added does not change the sum*.)

Commutative property of multiplication. If a and b are real numbers, then $ab = ba$. (*The order in which two numbers are multiplied does not change the product*.)

Composite number. A positive integer greater than 1 that is not a prime number.

Congruent. Having the same size and shape.

Dilation. A transformation in which a geometric figure is enlarged or reduced with respect to a given point (the center) and a given scale factor with corresponding lines in the original figure and the dilation being parallel.

Distributive property of multiplication over addition. If a and b are real numbers, then, $a(b + c) = ab + ac$, or $(a + b)c = ac + bc$.

Equilateral triangle. A triangle with all sides congruent.

Function. A rule that associates with each x-value in the domain exactly one y-value in the range.

Greatest common factor (GCF). The largest factor common to two or more numbers or terms.

Histogram. A graph representing the frequency of data using adjacent rectangular bars.

Identity property of addition. For all real numbers a, there exists a real number 0, such that $a + 0 = a$ and $0 + a = a$. The identity element for addition is 0.

Identity property of multiplication. For all real numbers a, there exists a real number 1, such that $a \cdot 1 = a$ and $1 \cdot a = a$. The identity element for multiplication is 1.

Integer. A whole number or its opposite.

Isosceles triangle. A triangle with two congruent sides.

Least common multiple (LCM). The smallest integer that is divisible by each of two or more integers.

Line symmetry. The property of a figure that identifies half of the figure as a reflection across a line of the other half of the figure.

Perpendicular bisector. A line, ray, or line segment that bisects a given line segment and forms a right angle with it.

Polygon. A simple, closed many-sided figure.

Prime number. An integer greater than 1, which has only itself, and 1 as positive divisors.

Proportion. An equation of two equal ratios.

Pythagorean Theorem. A relationship in a right triangle with legs a and b and hypotenuse c for which $a^2 + b^2 = c^2$.

Quadrilateral. A polygon with four sides.

Ratio. A comparison of two quantities using division.

Rational number. Any number that can be expressed as an integer, a ratio between two integers (where zero does not serve as the denominator), or a repeating or terminating decimal.

Regular polygon. A polygon with congruent sides and congruent angles.

Rotation. A transformation in which a geometric figure rotates around a point without affecting its size or shape.

Rotational symmetry. The property of a figure that identifies a rotation of the figure about a point as the same figure.

Scalene triangle. A triangle with no congruent sides.

Terminating decimal. A decimal number that ends in all zeros.

Transformation. Movement of a geometric figure from one location or size to another.

Translation. A transformation in which a geometric figure slides from one location to another without affecting its size and shape.

Transversal. A line that crosses other lines.

Triangle Inequality Theorem. The sum of the lengths of any two sides of a triangle must be greater than the length of the third side.

Vertex. The point common to both sides of an angle.